BIG IDEAS

The **concept of a limit** is foundational to calculus.	Differential calculus develops the concept of **instantaneous rate of change**.	Integral calculus develops the concept of determining a product involving a **continuously changing** quantity over an interval.	Derivatives and integrals are **inversely related**.

Learning Standards

Curricular Competencies	Content
Students are expected to do the following: **Reasoning and modelling** • Develop **thinking strategies** to solve puzzles and play games • Explore, **analyze**, and apply mathematical ideas using **reason**, **technology**, and **other tools** • **Estimate reasonably** and demonstrate **fluent, flexible, and strategic thinking** about number • **Model** with mathematics in **situational contexts** • **Think creatively** and with **curiosity and wonder** when exploring problems **Understanding and solving** • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, **inquiry**, and problem solving • **Visualize** to explore and illustrate mathematical concepts and relationships • Apply **flexible and strategic approaches** to **solve problems** • Solve problems with **persistence and a positive disposition** • Engage in problem-solving experiences **connected** with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures	*Students are expected to know the following:* • **functions** and graphs • **limits:** – left and right limits – limits to infinity – continuity • **differentiation:** – **rate of change** – **differentiation rules** – higher order, implicit – **applications** • **integration:** – **approximations** – fundamental theorem of calculus – **methods of integration** – **applications**

Learning Standards (continued)

Curricular Competencies	Content
Communicating and representing • **Explain and justify** mathematical ideas and **decisions** in **many ways** • **Represent** mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to **discussions** in the classroom • Take risks when offering ideas in classroom **discourse** **Connecting and reflecting** • **Reflect** on mathematical thinking • **Connect mathematical concepts** with each other, other areas, and personal interests • Use **mistakes** as **opportunities to advance learning** • **Incorporate** First Peoples worldviews, perspectives, **knowledge**, and **practices** to make connections with computer science concepts	

Big Ideas – Elaborations

- **concept of a limit:**
  - Differentiation and integration are defined using limits.

  Sample questions to support inquiry with students:
  - Why is a limit useful?
  - How can we use historical examples (e.g., Achilles and the tortoise) to describe a limit?
- **instantaneous rate of change:**
  - developing rate of change from average to instantaneous

  Sample questions to support inquiry with students:
  - How can a rate of change be instantaneous?
  - When do we use rate of change?
- **continuously changing:**
  - area (height x width) under a curve where the height of the region is changing; volume of a solid (area x length) where cross-sectional area is changing; work (force x distance) where force is changing
  - Finding these products requires finding an infinite sum.

  Sample questions to support inquiry with students:
  - What is the value of using rectangles to approximate the area under a curve?
  - Why is the fundamental theorem of calculus so fundamental?
- **inversely related:**
  - The fundamental theorem of calculus describes the relationship between integrals and antiderivatives.

  Sample questions to support inquiry with students:
  - How are derivatives and integrals related?
  - Why are antiderivatives important?
  - What is the difference between an antiderivative and an integral?

- **thinking strategies:**
  - using reason to determine winning strategies
  - generalizing and extending
- **analyze:**
  - examine the structure of and connections between mathematical ideas (e.g., limits, derivatives, integrals)
- **reason:**
  - inductive and deductive reasoning
  - predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding)
- **technology:**
  - graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps
  - can be used for a wide variety of purposes, including:
    - exploring and demonstrating mathematical relationships
    - organizing and displaying data
    - generating and testing inductive conjectures
    - mathematical modelling
- **other tools:**
  - manipulatives such as algebra tiles and other concrete materials
- **Estimate reasonably:**
  - be able to defend the reasonableness of an estimate across mathematical contexts
- **fluent, flexible, and strategic thinking:**
  - includes:
    - using known facts and benchmarks, partitioning, applying number strategies to approximate limits, derivatives, and integrals
    - choosing from different ways to think of a number or operation (e.g., Which will be the most strategic or efficient?)
- **Model:**
  - use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios)
  - take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it
- **situational contexts:**
  - including real-life scenarios and open-ended challenges that connect mathematics with everyday life
- **Think creatively:**
  - by being open to trying different strategies
  - refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music

- **curiosity and wonder:**
  - asking questions to further understanding or to open other avenues of investigation
- **inquiry:**
  - includes structured, guided, and open inquiry
  - noticing and wondering
  - determining what is needed to make sense of and solve problems
- **Visualize:**
  - create and use mental images to support understanding
  - Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations), concrete materials, drawings, and diagrams.
- **flexible and strategic approaches:**
  - deciding which mathematical tools to use to solve a problem
  - choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play)
- **solve problems:**
  - interpret a situation to identify a problem
  - apply mathematics to solve the problem
  - analyze and evaluate the solution in terms of the initial context
  - repeat this cycle until a solution makes sense
- **persistence and a positive disposition:**
  - not giving up when facing a challenge
  - problem solving with vigour and determination
- **connected:**
  - through daily activities, local and traditional practices, popular media and news events, cross-curricular integration
  - by posing and solving problems or asking questions about place, stories, and cultural practices
- **Explain and justify:**
  - using mathematical arguments to convince
  - includes anticipating consequences
- **decisions:**
  - Have students explore which of two scenarios they would choose and then defend their choice.
- **many ways:**
  - including oral, written, visual, use of technology
  - communicating effectively according to what is being communicated and to whom

- **Represent:**
  - using models, tables, graphs, words, numbers, symbols
  - connecting meanings among various representations
- **discussions:**
  - partner talks, small-group discussions, teacher-student conferences
- **discourse:**
  - is valuable for deepening understanding of concepts
  - can help clarify students' thinking, even if they are not sure about an idea or have misconceptions
- **Reflect:**
  - share the mathematical thinking of self and others, including evaluating strategies and solutions, extending, posing new problems and questions
- **Connect mathematical concepts:**
  - to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration)
- **mistakes:**
  - range from calculation errors to misconceptions
- **opportunities to advance learning:**
  - by:
    - analyzing errors to discover misunderstandings
    - making adjustments in further attempts
    - identifying not only mistakes but also parts of a solution that are correct
- **Incorporate:**
  - by:
    - collaborating with Elders and knowledge keepers among local First Peoples
    - exploring the First Peoples Principles of Learning (e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time)
    - making explicit connections with learning mathematics
    - exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections
- **knowledge:**
  - local knowledge and cultural practices that are appropriate to share and that are non-appropriated
- **practices:**
  - Bishop's cultural practices: counting, measuring, locating, designing, playing, explaining
  - Aboriginal Education Resources
  - *Teaching Mathematics in a First Nations Context*, FNESC

Content – Elaborations

- **functions:**
  - parent functions from Pre-calculus 12
  - piecewise functions
  - inverse trigonometric functions
- **limits:**
  - from table of values, graphically, and algebraically
  - one-sided versus two-sided
  - end behaviour
  - intermediate value theorem
- **differentiation:**
  - history
  - definition of derivative
  - notation
- **rate of change:**
  - average versus instantaneous
  - slope of secant and tangent lines
- **differentiation rules:**
  - power, product; quotient and chain
  - transcendental functions: logarithmic, exponential, trigonometric
- **applications:**
  - relating graph of $f(x)$ to $f'(x)$ and $f''(x)$
  - increasing/decreasing, concavity
  - differentiability, mean value theorem
  - Newton's method
  - problems in contextual situations, including related rates and optimization problems
- **integration:**
  - definition of an integral
  - notation
  - definite and indefinite

Content – Elaborations

- **approximations:**
  - Riemann sum, rectangle approximation method, trapezoidal method
- **methods of integration:**
  - antiderivatives of functions
  - substitution
  - by parts
- **applications:**
  - area under a curve, volume of solids, average value of functions
  - differential equations
  - initial value problems
  - slope fields