BIG IDEAS

Using **inverses** is the foundation of solving equations and can be extended to relationships between functions.	Understanding the characteristics of families of **functions** allows us to model and understand relationships and to build connections between classes of functions.	**Transformations** of shapes extend to functions and relations in all of their representations.

Learning Standards

Curricular Competencies	Content
Students are expected to do the following: **Reasoning and modelling** • Develop **thinking strategies** to solve puzzles and play games • Explore, **analyze**, and apply mathematical ideas using **reason**, **technology**, and **other tools** • **Estimate reasonably** and demonstrate **fluent, flexible, and strategic** thinking about number • **Model** with mathematics in **situational contexts** • **Think creatively** and with **curiosity and wonder** when exploring problems **Understanding and solving** • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, **inquiry**, and problem solving • **Visualize** to explore and illustrate mathematical concepts and relationships • Apply **flexible and strategic approaches** to **solve problems** • Solve problems with **persistence and a positive disposition** • Engage in problem-solving experiences **connected** with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures	*Students are expected to know the following:* • **transformations** of functions and relations • **exponential** functions and equations • **geometric** sequences and series • **logarithms:** operations, functions, and equations • **polynomial** functions and equations • **rational** functions • **trigonometry:** functions, equations, and identities

Learning Standards (continued)

Curricular Competencies	Content
Communicating and representing • **Explain and justify** mathematical ideas and **decisions** in **many ways** • **Represent** mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to **discussions** in the classroom • Take risks when offering ideas in classroom **discourse** **Connecting and reflecting** • **Reflect** on mathematical thinking • **Connect mathematical concepts** with each other, other areas, and personal interests • Use **mistakes** as **opportunities to advance learning** • **Incorporate** First Peoples worldviews, perspectives, **knowledge**, and **practices** to make connections with mathematical concepts	

- **inverses:**
  - *undo* the operations within an expression or function to reduce the expression to an identity (e.g., $x = $)

  Sample questions to support inquiry with students:
  - How can the inverse help to solve an equation?
  - How is solving an equation related to identifying the specific input for a function, given a specific output?
  - How are exponential and logarithmic functions related?
  - How are the laws of exponents connected to the laws of logarithms?
  - What are some other examples of inversely related functions?
  - How are inverses related graphically, and why?
  - How is solving an exponential equation similar to solving a trigonometric equation?
  - How are inverse operations related to solving a polynomial equation by factoring?
  - What is the value of using trigonometric identities to find equivalent expressions?
  - Why do some equations have extraneous roots and other equations do not?
- **functions:**

  Sample questions to support inquiry with students:
  - How do we decide which kind of function to use to model a given problem?
  - What do functions and relations look like beyond the visible axes?
  - A set of data looks like a parabola, but it is not. What function could be used to model this data?
  - What does the number of zeros tell us about a function?
  - What connections do we see within the characteristics of a particular class of function?
- **Transformations:**

  Sample questions to support inquiry with students:
  - How can we tell whether a transformation will have invariant points?
  - Under what circumstances will different transformations produce the same result?
  - How do graphical transformations affect the tables of values?
  - How does a transformation affect a point found at the origin as compared to a point on an axis or a point in one of the four quadrants?
  - How can a rational function of the form $y = \frac{ax+b}{cx+d}$ be considered as a transformation of the reciprocal function $y = \frac{1}{x}$?

- **thinking strategies:**
  - using reason to determine winning strategies
  - generalizing and extending
- **analyze:**
  - examine the structure of and connections between mathematical ideas (e.g., exponential functions to geometric sequences)
- **reason:**
  - inductive and deductive reasoning
  - predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding)
- **technology:**
  - graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps
  - can be used to for a wide variety of purposes, including:
    - exploring and demonstrating mathematical relationships
    - organizing and displaying data
    - generating and testing inductive conjectures
    - mathematical modelling
- **other tools:**
  - manipulatives such as algebra tiles and other concrete materials
- **Estimate reasonably:**
  - be able to defend the reasonableness of an estimated value or a solution to a problem or equation (e.g., the zeros of a graphed polynomial function)
- **fluent, flexible and strategic thinking:**
  - includes:
    - using known facts and benchmarks, partitioning, applying whole number strategies to rational numbers and algebraic expressions
    - choosing from different ways to think of a number or operation (e.g., Which will be the most strategic or efficient?)
- **Model:**
  - use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios)
  - take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it
- **situational contexts:**
  - including real-life scenarios and open-ended challenges that connect mathematics with everyday life
- **Think creatively:**
  - by being open to trying different strategies
  - refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music

- **curiosity and wonder:**
  - asking questions to further understanding or to open other avenues of investigation
- **inquiry:**
  - includes structured, guided, and open inquiry
  - noticing and wondering
  - determining what is needed to make sense of and solve problems
- **Visualize:**
  - create and use mental images to support understanding
  - Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations), concrete materials, drawings, and diagrams.
- **flexible and strategic approaches:**
  - deciding which mathematical tools to use to solve a problem
  - choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play)
- **solve problems:**
  - interpret a situation to identify a problem
  - apply mathematics to solve the problem
  - analyze and evaluate the solution in terms of the initial context
  - repeat this cycle until a solution makes sense
- **persistence and a positive disposition:**
  - not giving up when facing a challenge
  - problem solving with vigour and determination
- **connected:**
  - through daily activities, local and traditional practices, popular media and news events, cross-curricular integration
  - by posing and solving problems or asking questions about place, stories, and cultural practices
- **Explain and justify:**
  - use mathematical arguments to convince
  - includes anticipating consequences
- **decisions:**
  - Have students explore which of two scenarios they would choose and then defend their choice.
- **many ways:**
  - including oral, written, visual, use of technology

- **Represent:**
  - using models, tables, graphs, words, numbers, symbols
  - connecting meanings among various representations
- **discussions:**
  - partner talks, small-group discussions, teacher-student conferences
- **discourse:**
  - is valuable for deepening understanding of concepts
  - can help clarify students' thinking, even if they are not sure about an idea or have misconceptions
- **Reflect:**
  - share the mathematical thinking of self and others, including evaluating strategies and solutions, extending, posing new problems and questions
- **Connect mathematical concepts:**
  - to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration)
- **mistakes:**
  - range from calculation errors to misconceptions
- **opportunities to advance learning:**
  - by:
    - analyzing errors to discover misunderstandings
    - making adjustments in further attempts
    - identifying not only mistakes but also parts of a solution that are correct
- **Incorporate:**
  - by:
    - collaborating with Elders and knowledge keepers among local First Peoples
    - exploring the First Peoples Principles of Learning (e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time)
    - making explicit connections with learning mathematics
    - exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections
- **knowledge:**
  - local knowledge and cultural practices that are appropriate to share and that are non-appropriated
- **practices:**
  - Bishop's cultural practices: counting, measuring, locating, designing, playing, explaining
  - Aboriginal Education Resources
  - *Teaching Mathematics in a First Nations Context*, FNESC

Content – Elaborations

- **transformations:**
  - of graphs and equations of parent functions and relations (e.g., absolute value, radical, reciprocal, conics, exponential, logarithmic, trigonometric)
  - vertical and horizontal translations, stretches, and reflections
  - inverses: graphs and equations
  - extension:
    - recognizing composed functions (e.g., $y =$)
    - operations on functions
- **exponential:**
  - graphing, including transformations
  - solving equations with same base and with different bases, including base *e*
  - solving problems in situational contexts
- **geometric:**
  - common ratio, first term, general term
  - geometric sequences connecting to exponential functions
  - infinite geometric series
  - sigma notation
- **logarithms:**
  - applying laws of logarithms
  - evaluating with different bases
  - using common and natural logarithms
  - exploring inverse of exponential
  - graphing, including transformations
  - solving equations with same base and with different bases
  - solving problems in situational contexts
- **polynomial:**
  - factoring, including the factor theorem and the remainder theorem
  - graphing and the characteristics of a graph (e.g., degree, extrema, zeros, end-behaviour)
  - solving equations algebraically and graphically
- **rational:**
  - characteristics of graphs, including asymptotes, intercepts, point discontinuities, domain, end-behaviour

Content – Elaborations

- **trigonometry:**
  - examining angles in standard position in both radians and degrees
  - exploring unit circle, reference and coterminal angles, special angles
  - graphing primary trigonometric functions, including transformations and characteristics
  - solving first- and second-degree equations (over restricted domains and all real numbers)
  - solving problems in situational contexts
  - using identities to reduce complexity in expressions and solve equations (e.g., Pythagorean, quotient, double angle, reciprocal, sum and difference)