

BIG IDEAS

<p>The concept of a limit is foundational to calculus.</p>	<p>Differential calculus develops the concept of instantaneous rate of change.</p>	<p>Integral calculus develops the concept of determining a product involving a continuously changing quantity over an interval.</p>	<p>Derivatives and integrals are inversely related.</p>
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Learning Standards

Curricular Competencies	Content
<p><i>Students are expected to do the following:</i></p> <p>Reasoning and modelling</p> <ul style="list-style-type: none"> • Develop thinking strategies to solve puzzles and play games • Explore, analyze, and apply mathematical ideas using reason, technology, and other tools • Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about number • Model with mathematics in situational contexts • Think creatively and with curiosity and wonder when exploring problems <p>Understanding and solving</p> <ul style="list-style-type: none"> • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through experimentation, inquiry, and problem solving • Visualize to explore and illustrate mathematical concepts and relationships • Apply flexible and strategic approaches to solve problems • Solve problems with persistence and a positive disposition • Engage in problem-solving experiences connected with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures 	<p><i>Students are expected to know the following:</i></p> <ul style="list-style-type: none"> • functions and graphs • limits: <ul style="list-style-type: none"> – left and right limits – limits to infinity – continuity • differentiation: <ul style="list-style-type: none"> – rate of change – differentiation rules – higher order, implicit – applications • integration: <ul style="list-style-type: none"> – approximations – fundamental theorem of calculus – methods of integration – applications

Learning Standards (continued)

Curricular Competencies	Content
<p>Communicating and representing</p> <ul style="list-style-type: none"> • Explain and justify mathematical ideas and decisions in many ways • Represent mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to discussions in the classroom • Take risks when offering ideas in classroom discourse <p>Connecting and reflecting</p> <ul style="list-style-type: none"> • Reflect on mathematical thinking • Connect mathematical concepts with each other, other areas, and personal interests • Use mistakes as opportunities to advance learning • Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with computer science concepts 	

Big Ideas – Elaborations

• **concept of a limit:**

- Differentiation and integration are defined using limits.

Sample questions to support inquiry with students:

- Why is a limit useful?
- How can we use historical examples (e.g., Achilles and the tortoise) to describe a limit?

• **instantaneous rate of change:**

- developing rate of change from average to instantaneous

Sample questions to support inquiry with students:

- How can a rate of change be instantaneous?
- When do we use rate of change?

• **continuously changing:**

- area (height x width) under a curve where the height of the region is changing; volume of a solid (area x length) where cross-sectional area is changing; work (force x distance) where force is changing
- Finding these products requires finding an infinite sum.

Sample questions to support inquiry with students:

- What is the value of using rectangles to approximate the area under a curve?
- Why is the fundamental theorem of calculus so fundamental?

• **inversely related:**

- The fundamental theorem of calculus describes the relationship between integrals and antiderivatives.

Sample questions to support inquiry with students:

- How are derivatives and integrals related?
- Why are antiderivatives important?
- What is the difference between an antiderivative and an integral?

Curricular Competencies – Elaborations

- **thinking strategies:**
 - using reason to determine winning strategies
 - generalizing and extending
- **analyze:**
 - examine the structure of and connections between mathematical ideas (e.g., limits, derivatives, integrals)
- **reason:**
 - inductive and deductive reasoning
 - predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding)
- **technology:**
 - graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps
 - can be used for a wide variety of purposes, including:
 - exploring and demonstrating mathematical relationships
 - organizing and displaying data
 - generating and testing inductive conjectures
 - mathematical modelling
- **other tools:**
 - manipulatives such as algebra tiles and other concrete materials
- **Estimate reasonably:**
 - be able to defend the reasonableness of an estimate across mathematical contexts
- **fluent, flexible, and strategic thinking:**
 - includes:
 - using known facts and benchmarks, partitioning, applying number strategies to approximate limits, derivatives, and integrals
 - choosing from different ways to think of a number or operation (e.g., Which will be the most strategic or efficient?)
- **Model:**
 - use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios)
 - take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it
- **situational contexts:**
 - including real-life scenarios and open-ended challenges that connect mathematics with everyday life
- **Think creatively:**
 - by being open to trying different strategies
 - refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music

Curricular Competencies – Elaborations

- **curiosity and wonder:**
 - asking questions to further understanding or to open other avenues of investigation
- **inquiry:**
 - includes structured, guided, and open inquiry
 - noticing and wondering
 - determining what is needed to make sense of and solve problems
- **Visualize:**
 - create and use mental images to support understanding
 - Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations), concrete materials, drawings, and diagrams.
- **flexible and strategic approaches:**
 - deciding which mathematical tools to use to solve a problem
 - choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play)
- **solve problems:**
 - interpret a situation to identify a problem
 - apply mathematics to solve the problem
 - analyze and evaluate the solution in terms of the initial context
 - repeat this cycle until a solution makes sense
- **persistence and a positive disposition:**
 - not giving up when facing a challenge
 - problem solving with vigour and determination
- **connected:**
 - through daily activities, local and traditional practices, popular media and news events, cross-curricular integration
 - by posing and solving problems or asking questions about place, stories, and cultural practices
- **Explain and justify:**
 - using mathematical arguments to convince
 - includes anticipating consequences
- **decisions:**
 - Have students explore which of two scenarios they would choose and then defend their choice.
- **many ways:**
 - including oral, written, visual, use of technology
 - communicating effectively according to what is being communicated and to whom

Curricular Competencies – Elaborations

- **Represent:**
 - using models, tables, graphs, words, numbers, symbols
 - connecting meanings among various representations
- **discussions:**
 - partner talks, small-group discussions, teacher-student conferences
- **discourse:**
 - is valuable for deepening understanding of concepts
 - can help clarify students’ thinking, even if they are not sure about an idea or have misconceptions
- **Reflect:**
 - share the mathematical thinking of self and others, including evaluating strategies and solutions, extending, posing new problems and questions
- **Connect mathematical concepts:**
 - to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration)
- **mistakes:**
 - range from calculation errors to misconceptions
- **opportunities to advance learning:**
 - by:
 - analyzing errors to discover misunderstandings
 - making adjustments in further attempts
 - identifying not only mistakes but also parts of a solution that are correct
- **Incorporate:**
 - by:
 - collaborating with Elders and knowledge keepers among local First Peoples
 - exploring the [First Peoples Principles of Learning](#) (e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time)
 - making explicit connections with learning mathematics
 - exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections
- **knowledge:**
 - local knowledge and cultural practices that are appropriate to share and that are non-appropriated
- **practices:**
 - [Bishop’s cultural practices](#): counting, measuring, locating, designing, playing, explaining
 - [Aboriginal Education Resources](#)
 - [Teaching Mathematics in a First Nations Context](#), FNECS

Content – Elaborations

- **functions:**
 - parent functions from Pre-calculus 12
 - piecewise functions
 - inverse trigonometric functions
- **limits:**
 - from table of values, graphically, and algebraically
 - one-sided versus two-sided
 - end behaviour
 - intermediate value theorem
- **differentiation:**
 - history
 - definition of derivative
 - notation
- **rate of change:**
 - average versus instantaneous
 - slope of secant and tangent lines
- **differentiation rules:**
 - power, product; quotient and chain
 - transcendental functions: logarithmic, exponential, trigonometric
- **applications:**
 - relating graph of $f(x)$ to $f'(x)$ and $f''(x)$
 - increasing/decreasing, concavity
 - differentiability, mean value theorem
 - Newton's method
 - problems in contextual situations, including related rates and optimization problems
- **integration:**
 - definition of an integral
 - notation
 - definite and indefinite

Content – Elaborations

- **approximations:**
 - Riemann sum, rectangle approximation method, trapezoidal method
- **methods of integration:**
 - antiderivatives of functions
 - substitution
 - by parts
- **applications:**
 - area under a curve, volume of solids, average value of functions
 - differential equations
 - initial value problems
 - slope fields