

## BIG IDEAS

**Diagrams** are fundamental to investigating, communicating, and discovering properties and relations in geometry.

Finding **invariance amidst variation** drives geometric investigation.

Geometry involves creating, testing, and refining **definitions**.

The **proving process** begins with conjecturing, looking for counter-examples, and refining the conjecture, and the process may end with a written proof.

**Geometry** stories and applications vary across cultures and time.

## Learning Standards

Curricular Competencies	Content
<p><i>Students are expected to do the following:</i></p> <p><b>Reasoning and modelling</b></p> <ul style="list-style-type: none"> <li>• Develop <b>thinking strategies</b> to solve puzzles and play games</li> <li>• Engage in <b>spatial reasoning</b> in a dynamic environment</li> <li>• Explore, <b>analyze</b>, and apply mathematical ideas using <b>reason</b>, <b>technology</b>, and <b>other tools</b></li> <li>• <b>Estimate reasonably</b> and demonstrate <b>fluent, flexible, and strategic thinking</b> about number</li> <li>• <b>Model</b> with mathematics in <b>situational contexts</b></li> <li>• <b>Think creatively</b> and with <b>curiosity and wonder</b> when exploring problems</li> </ul> <p><b>Understanding and solving</b></p> <ul style="list-style-type: none"> <li>• Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, <b>inquiry</b>, and problem solving</li> <li>• <b>Visualize</b> to explore and illustrate geometric concepts and relationships</li> <li>• Apply <b>flexible and strategic approaches</b> to <b>solve problems</b></li> <li>• Solve problems with <b>persistence and a positive disposition</b></li> <li>• Engage in problem-solving experiences <b>connected</b> with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures</li> </ul>	<p><i>Students are expected to know the following:</i></p> <ul style="list-style-type: none"> <li>• geometric <b>constructions</b></li> <li>• <b>parallel and perpendicular</b> lines: <ul style="list-style-type: none"> <li>– <b>circles as tools</b> in constructions</li> <li>– perpendicular bisector</li> </ul> </li> <li>• <b>circle geometry</b></li> <li>• <b>constructing tangents</b></li> <li>• transformations of 2D shapes: <ul style="list-style-type: none"> <li>– <b>isometries</b></li> <li>– <b>non-isometric transformations</b></li> </ul> </li> <li>• <b>non-Euclidean geometries</b></li> </ul>

Learning Standards (continued)

Curricular Competencies	Content
<p><b>Communicating and representing</b></p> <ul style="list-style-type: none"> <li>• <b>Explain, justify</b>, and evaluate geometric ideas and <b>decisions</b> in <b>many ways</b></li> <li>• <b>Represent</b> mathematical ideas in concrete, pictorial, and symbolic forms</li> <li>• Use geometric vocabulary and language to contribute to <b>discussions</b> in the classroom</li> <li>• Take risks when offering ideas in classroom <b>discourse</b></li> </ul> <p><b>Connecting and reflecting</b></p> <ul style="list-style-type: none"> <li>• <b>Reflect</b> on geometric thinking</li> <li>• <b>Connect mathematical concepts</b> with each other, other areas, and personal interests</li> <li>• Use <b>mistakes</b> as <b>opportunities to advance learning</b></li> <li>• <b>Incorporate</b> First Peoples worldviews, perspectives, <b>knowledge</b>, and <b>practices</b> to make connections with computer science concepts</li> </ul>	

Big Ideas – Elaborations	MATHEMATICS – Geometry Grade 12
<ul style="list-style-type: none"> <li>• <b>Diagrams:</b>  <i>Sample questions to support inquiry with students:</i> <ul style="list-style-type: none"> <li>– How would we describe a specific geometric object to someone who cannot see it?</li> <li>– What properties can we infer from a diagram?</li> <li>– What behaviours can we infer from a dynamic diagram?</li> </ul> </li> </ul>	

**Big Ideas – Elaborations**

• **invariance amidst variation:**

- Invariance amidst variation can be more easily experienced using current technology and dynamic diagrams. For example, the sum of the angles in planar triangles is invariant no matter what forms a triangle takes.

*Sample questions to support inquiry with students:*

- How do we construct geometric shapes that maintain properties under variation?
- What properties change and stay the same when we vary a square, parallelogram, triangle, and so on?
- How can the Pythagorean theorem be restated in terms of variance and invariance?

• **definitions:**

- are seldom the starting point in geometry

*Sample questions to support inquiry with students:*

- How does variation help to refine our definitions of shapes?
- How would we define a square (or a circle) in different ways? When would one definition be better to work with than another?
- How can the definition of a shape be used in constructing the shape?
- How can we modify a definition of a shape to define a new shape?

• **proving process:**

*Sample questions to support inquiry with students:*

- Can we make a conjecture about the diagonals of a polygon? Can we find a counter-example to our conjecture?
- How can one conjecture about a *specific* shape lead to making another more *general* conjecture about a family of shapes?
- How can we be sure that a proof is complete?
- Can we find a counter-example to a conjecture?
- How can different proofs bring out different understandings of a relationship?

• **Geometry:**

- Geometry is more than a list of axioms and deductions. Non-Western and modern geometry is concerned with shape and space and is not always axiomatic. It is not always about producing a theorem; rather, it is about modelling mathematical and non-mathematical phenomena using geometric objects and relations. Today geometry is used in a multitude of disciplines, including animation, architecture, biology, carpentry, chemistry, medical imaging, and art.

*Sample questions to support inquiry with students:*

- Can we find geometric relationships in local First Peoples art or culture?
- Can we make geometric connections to story, language, or past experiences?
- What do we notice about and how would we construct common shapes found in local First Peoples art?
- How has the notion of “proof” changed over time and in different cultures?
- How are geometric ideas implemented in modern professions?

Curricular Competencies – Elaborations

- **thinking strategies:**
  - using reason to determine winning strategies
  - generalizing and extending
- **spatial reasoning:**
  - being able to think about shapes (real or imagined) and mentally transform them to notice relationships
- **analyze:**
  - examine the structure of and connections between geometric ideas (e.g., parallel and perpendicular lines, circle geometry, constructing tangents, transformations)
- **reason:**
  - inductive and deductive reasoning
  - predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding)
- **technology:**
  - graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps
  - can be used for a wide variety of purposes, including:
    - exploring and demonstrating geometrical relationships
    - organizing and displaying data
    - generating and testing inductive conjectures
    - mathematical modelling
- **other tools:**
  - paper and scissors, straightedge and compass, ruler, and other concrete materials
- **Estimate reasonably:**
  - be able to defend the reasonableness of an estimated value or a solution to a problem or equation (e.g., congruencies, angles, lengths)
- **fluent, flexible, and strategic thinking:**
  - being able to generate a family of shapes and apply characteristics across the family
- **Model:**
  - use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios)
  - take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it
- **situational contexts:**
  - including real-life scenarios and open-ended challenges that connect mathematics with everyday life
- **Think creatively:**
  - by being open to trying different strategies
  - refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music

Curricular Competencies – Elaborations

- **curiosity and wonder:**
  - asking questions to further understanding or to open other avenues of investigation
- **inquiry:**
  - includes structured, guided, and open inquiry
  - noticing and wondering
  - determining what is needed to make sense of and solve problems
- **Visualize:**
  - create and use mental images to support understanding
  - Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations), concrete materials, drawings, and diagrams.
- **flexible and strategic approaches:**
  - deciding which mathematical tools to use to solve a problem
  - choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play)
- **solve problems:**
  - interpret a situation to identify a problem
  - apply mathematics to solve the problem
  - analyze and evaluate the solution in terms of the initial context
  - repeat this cycle until a solution makes sense
- **persistence and a positive disposition:**
  - not giving up when facing a challenge
  - problem solving with vigour and determination
- **connected:**
  - through daily activities, local and traditional practices, popular media and news events, cross-curricular integration
  - by posing and solving problems or asking questions about place, stories, and cultural practices
- **Explain and justify:**
  - use geometrical arguments to convince
  - includes anticipating consequences
- **decisions:**
  - Have students explore which of two scenarios they would choose and then defend their choice.
- **many ways:**
  - including oral, written, visual, gestures use of technology
  - communicating effectively according to what is being communicated and to whom

## Curricular Competencies – Elaborations

- **Represent:**
  - concretely, diagrammatically, symbolically, including using models, tables, graphs, words, numbers, symbols
- **discussions:**
  - partner talks, small-group discussions, teacher-student conferences
- **discourse:**
  - is valuable for deepening understanding of concepts
  - can help clarify students’ thinking, even if they are not sure about an idea or have misconceptions
- **Reflect**
  - share the geometric thinking of self and others, including evaluating strategies and solutions, finding counter-examples, extending, posing new problems and questions, proving results
- **Connect mathematical concepts:**
  - to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration)
- **mistakes:**
  - range from calculation errors to misconceptions
- **opportunities to advance learning:**
  - by:
    - analyzing errors to discover misunderstandings
    - making adjustments in further attempts
    - identifying not only mistakes but also parts of a solution that are correct
- **Incorporate:**
  - by:
    - collaborating with Elders and knowledge keepers among local First Peoples
    - exploring the [First Peoples Principles of Learning](#) (e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time)
    - making explicit connections with learning mathematics
    - exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections
- **knowledge:**
  - local knowledge and cultural practices that are appropriate to share and that are non-appropriated
- **practices:**
  - [Bishop’s cultural practices](#): counting, measuring, locating, designing, playing, explaining
  - [Aboriginal Education Resources](#)
  - [Teaching Mathematics in a First Nations Context](#), FNECS

Content – Elaborations

- **constructions:**
  - angles, triangles, triangle centres, quadrilaterals
- **parallel and perpendicular:**
  - angle bisector
- **circles as tools:**
  - constructing equal segments, midpoints
- **circle geometry:**
  - properties of chords, angles, and tangents to mobilize the proving process
- **constructing tangents:**
  - lines tangent to circles, circles tangent to circles, circles tangent to three objects (e.g., points [PPP], three lines [LLL])
- **isometries:**
  - transformations that maintain congruence (translations, rotations, reflections)
  - composition of isometries
  - tessellations
- **non-isometric transformations:**
  - dilations and shear
  - topology
- **non-Euclidean geometries:**
  - perspective, spherical, Taxicab, hyperbolic
  - tessellations