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Rubric and Elaborations

**Numeracy Assessment Rubric**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Snapshot</strong></td>
<td>The student demonstrates an inadequate understanding of the situation. The strategy is ineffective. The solution may contain fundamental mathematical errors. The reasoning is missing or irrelevant; the logic does not reference the problem.</td>
<td>The student demonstrates a basic understanding of the situation. The strategy is unclear and/or incomplete. The solution may contain mathematical errors. The reasoning is unclear; but the logic correctly references some aspects of the problem.</td>
<td>The student demonstrates an adequate understanding of the situation. The strategy is sensible but has some inconsistencies. The solution may contain minor mathematical errors. The reasoning is evident, and the logic references most aspects of the problem.</td>
<td>The student demonstrates a proficient understanding of the situation. The strategy is effective and comprehensive. The solution may contain minor mathematical errors that do not affect the demonstration of proficiency. The reasoning is clear and the logic references all aspects of the problem.</td>
</tr>
<tr>
<td>NR</td>
<td>No response (answer page is blank).</td>
<td>0</td>
<td>Information simply recopied from the problem. Diagrams or calculations are unrelated to the problem. Response does not address the purpose of the task. An incorrect mathematical solution with no work shown. Inappropriate response (contains profanity, inappropriate diagram or language). All work is erased or crossed out. Any zero score must include rationale and be approved by the section head.</td>
<td></td>
</tr>
</tbody>
</table>

**Note:**

All rationales correspond to the revised rubric snapshot shown above. The rubric is used for holistic scoring of all extensive constructed response questions. For each exemplar in the document, the descriptors identified from the rubric are the best representation for that exemplar. The bullet points underneath describe, in whole or in part, the evidence found within the student work, but do not form a complete list of what is needed for that particular score, and therefore should not be used as such.
<table>
<thead>
<tr>
<th>Elaborations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interpret</strong></td>
</tr>
<tr>
<td>• Demonstrates limited reasoning skills in interpreting the situation.</td>
</tr>
<tr>
<td>• Makes fundamental errors when making decisions about relevant information given in text, symbols, or graphical forms.</td>
</tr>
<tr>
<td>• Demonstrates effectiveness reasoning skills in interpreting the situation.</td>
</tr>
<tr>
<td>• Makes some errors when making decisions about relevant information given in text, symbols, or graphical forms.</td>
</tr>
<tr>
<td>• Demonstrates effective reasoning skills in interpreting the situation.</td>
</tr>
<tr>
<td>• Makes minor errors when making decisions about relevant information given in text, symbols, or graphical forms.</td>
</tr>
<tr>
<td>• Demonstrates advanced reasoning skills in interpreting the situation.</td>
</tr>
<tr>
<td>• Makes no errors when making decisions about relevant information given in text, symbols, or graphical forms.</td>
</tr>
</tbody>
</table>

Rubric and elaborations will be further refined following the first assessment administration and will be available in the spring of 2018.
A newspaper headline grabs your attention.

**DAILY NEWS**

**Water use skyrockets**

Recent studies have predicted water shortages.

![Bar graph showing average water use (L/person/day) for various locations: Cranbrook, Fort St. John, Fruitvale, Kamloops, Lillooet, Port Alberni, Prince George, Vancouver, Whistler.]

**Indoor Water Use with Conventional Appliances and Fixtures**

- **Toilet**: 13 L/flush
- **Tap**: 8 L/minute
- **Shower**: 10 L/minute
- **Dishwasher**: 40 L/cycle
- **Bath**: 100 L/bath
- **Leaks**: 3 L/day
- **Clothes Washer**: 150 L/cycle
High-efficiency appliances and fixtures can help reduce the amount of water we use.

Indoor Water Use with High-Efficiency Appliances and Fixtures

- Toilet 6 L/flush
- Shower 8 L/minute
- Dishwasher 16 L/cycle
- Tap 6 L/minute
- Bath 50 L/bath
- Clothes Washer 60 L/cycle
13. You want to reduce your personal water use to 1050 L/week. You install high-efficiency appliances and fixtures, and change your water-use habits.

Plan a water budget for yourself for 1 week that meets this goal using the high-efficiency appliances and fixtures.

Explain and justify your solution.

You must use everything in the table below at least once in the week.

<table>
<thead>
<tr>
<th>High-Efficiency Appliances and Fixtures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shower and/or bath</td>
</tr>
<tr>
<td>Toilet</td>
</tr>
<tr>
<td>Tap</td>
</tr>
<tr>
<td>Dishwasher</td>
</tr>
<tr>
<td>Clothes Washer</td>
</tr>
</tbody>
</table>

This question is to be answered on paper.

**Summary of Requirements for a Level 4:**

- Use less than 1050 L/week
- Use each of the items in the table (shower or bath, toilet, tap, dishwasher, clothes washer) at least once
- Be realistic (e.g., cannot flush toilet once per week)
- Can do a daily budget and then show that it is being replicated daily for the week, or can do a full seven-day budget with water usage varying each day
- Provide evidence to justify budget
- Communicate solution in context
There are many possible solutions; a couple of solutions are given.

**Solution 1**

**Daily Water Usage**

<table>
<thead>
<tr>
<th>Activity/Appliance</th>
<th>Daily Usage</th>
<th>Total</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shower</td>
<td>5 min/day 5 × 8 L/min = 40 L</td>
<td>40 L</td>
<td>One quick shower per day is all that is needed</td>
</tr>
<tr>
<td>Toilet</td>
<td>3 flushes/day 3 × 6 L = 18 L</td>
<td>18 L</td>
<td>Assuming you work or go to school during the day you only average using the toilet 3×/day</td>
</tr>
<tr>
<td>Tap</td>
<td>10 min/day 10 × 6 L/min = 60 L</td>
<td>60 L</td>
<td>After using washroom, and once for dishes that can't go in dishwasher</td>
</tr>
<tr>
<td>Dishwasher</td>
<td>Once every second day</td>
<td>8 L</td>
<td>One load of dishes per two days</td>
</tr>
<tr>
<td>Clothes Washer</td>
<td>Two loads/week (/7)</td>
<td>17 L</td>
<td>Only use a light and dark load per week</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td></td>
<td>143 L/day</td>
<td></td>
</tr>
</tbody>
</table>

143 L/day × 7 days = 1001 L for the week, which is under the limit of 1050 L per week.
Solution 2

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shower/bath</td>
<td>5 min shower</td>
<td>5 min shower</td>
<td>5 min shower</td>
<td>5 min shower</td>
<td>5 min shower</td>
<td>5 min shower</td>
<td>5 min shower</td>
</tr>
<tr>
<td></td>
<td>5 min × 8</td>
<td>5 min × 8</td>
<td>5 min × 8</td>
<td>5 min × 8</td>
<td>5 min × 8</td>
<td>5 min × 8</td>
<td>5 min × 8</td>
</tr>
<tr>
<td></td>
<td>L/min = 40 L</td>
<td>L/min = 40 L</td>
<td>L/min = 40 L</td>
<td>L/min = 40 L</td>
<td>L/min = 40 L</td>
<td>L/min = 40 L</td>
<td>L/min = 40 L</td>
</tr>
<tr>
<td>Toilet</td>
<td>3 × 6 L = 18 L</td>
<td>3 × 6 L = 18 L</td>
<td>3 × 6 L = 18 L</td>
<td>3 × 6 L = 18 L</td>
<td>5 × 6 L = 30 L</td>
<td>5 × 6 L = 30 L</td>
<td></td>
</tr>
<tr>
<td>Tap</td>
<td>8 min × 6</td>
<td>10 min × 6</td>
<td>8 min × 6</td>
<td>10 min × 6</td>
<td>10 min × 6</td>
<td>10 min × 6</td>
<td>8 min × 6</td>
</tr>
<tr>
<td></td>
<td>L/min = 48 L</td>
<td>L/min = 60 L</td>
<td>L/min = 48 L</td>
<td>L/min = 60 L</td>
<td>L/min = 60 L</td>
<td>L/min = 60 L</td>
<td>L/min = 48 L</td>
</tr>
<tr>
<td>Dishwasher</td>
<td>1 × 16 L = 16 L</td>
<td>1 × 16 L = 16 L</td>
<td>1 × 16 L = 16 L</td>
<td>1 × 16 L = 16 L</td>
<td>1 × 16 L = 16 L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clothes washer</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1 cycle = 60 L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>122L</td>
<td>178L</td>
<td>122L</td>
<td>118L</td>
<td>122L</td>
<td>190L</td>
<td>134L</td>
</tr>
</tbody>
</table>

Weekly total: 122 + 178 + 122 + 118 + 122+ 190 + 134 = 986 L
Weekly total is 986 L which is well under the total of 1050 L per week needed. This leaves room for any unexpected water usage.

Showers were taken daily. Toilet usage was increased on the weekend as I would not be using the facility at school as I do during the week.

The dishwasher is used every second day, so on the days it doesn’t run there are an extra 2 minutes of tap usage to accommodate having to wash a couple of dishes by hand.

Two loads of clothes washing per week are enough to have one load of lights and one load of darks. This keeps the water usage under the limit of 1050 L per week.
Exemplar #1 – Score: 4

The work demonstrates a proficient understanding of the situation.
• water usage is identified and explained
• all work is shown

The strategy is effective and comprehensive.
• calculations are clear and correct for each appliance/fixture

The logic references all aspects of the problem.
• each fixture use has a reasonable explanation
• each fixture is mentioned

The reasoning is clear, detailed, and organized.
• the total water usage is within the allowable amount
• communication is clear

Show your work and write your final answer in the space provided.

Shower: take around 8 minutes so 8L x 8 min = 64L every shower. Taking a shower every 2 days in a week would be 3-4 days of showering.
64L x 3 days = 192L every week.

Dishwasher: used a few times every week so 16L x 3 = 48L every week.

Toilet: 4 L per use x 7 days = 28L each week.

The tap is used around 10 minutes each day so 6L x 10 min = 60 then multiplied by 7 to get 420L every week.

Clothes Washer: is turned 2-3 times each week so 60L x 3
= 180L each week.

The total amount of water used is 1008L/week.
Exemplar #2 – Score: 4

- The work demonstrates a proficient understanding of the situation.
  - water usage is identified and explained
  - all work is shown

- The strategy is effective and comprehensive.
  - calculations are per fixture per week
  - calculations are clear and correct for each appliance/fixture

- The logic references all aspects of the problem.
  - each fixture is mentioned

- The reasoning is clear, detailed, and organized.
  - the total water usage is within the allowable amount
  - communication is clear
Exemplar #3 – Score: 3

☑️ The work demonstrates an adequate understanding of the situation.
- water usage is identified
- work is shown

☑️ The strategy is sensible but has some inconsistencies.
- most fixtures are calculated per day x 7 for the week
- use of dishwasher and bath weekly total is not explained

☑️ The logic references all aspects of the problem.
- each fixture is used

☑️ The reasoning is evident.
- most supporting calculations are shown
- water usage is within the allowable amount

\[
I_w = 7 \times 150 = 1050\text{L} \\
5 \text{ Flushes a day} = 30\text{L} \times 7 = 210\text{L} \\
10\text{min. tap} \text{ a day} = 60 \times 7 = 420\text{L} \\
5 \text{ showers} 5\text{ min} = 40 \times 5 = 200\text{L} \\
1 \text{ load of laundry} = 60\text{L} \\
\text{Dishwasher per day} = 112\text{L} \\
\text{Bath} = 50\text{L} \\
= 1052\text{L}
\]
Exemplar #4 – Score: 3

- The work demonstrates an adequate understanding of the situation.
  - end result meets the criteria

- The strategy is sensible but has some inconsistencies.
  - appropriate strategy is used but the daily consumption does not match the weekly consumption

- The logic references most aspects of the problem.
  - incomplete justification (not all calculations are shown)
  - running the dishwasher and clothes washer once a day is not logical

- The reasoning is evident.
  - most supporting calculations are shown
  - communication is clear and organized

```
By weekday:
4 min a day = 3L a day
2 flushes a day = 1L
5 min a day = 30L a day
1 cycle a day = 14L
1 cycle a day = 10L

Why frequencies chosen:
- one shower a day for only 4 minutes
- avg. person only needs to flush the toilet twice
- the bathroom tap can be conserved to 2 minutes leaving 3 for the kitchen
- the dishwasher only needs to be run at the end of the day
- the laundry needs to be done at the end of the day

Taps (bathroom and kitchen):

Overall consumption in a day = 150
Overall consumption in a week = 1050
```
Exemplar #5 – Score: 2

- The work demonstrates a basic understanding of the situation.
  - incomplete mathematical solution
- The strategy is unclear and/or incomplete.
  - appropriate strategy selected, but not followed through
- The logic correctly references some aspects of the problem.
  - all fixtures are included, but total use is not calculated
- The reasoning is clear but incomplete.
  - supporting calculations are not shown
  - the work shown has some structure but no real solution is presented

Image:

```
<table>
<thead>
<tr>
<th>Day</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shower/ Bath</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Toilet</td>
<td>18l/3 flush</td>
<td>3 flush</td>
<td>3 flush</td>
<td>3 flush</td>
<td>3 flush</td>
<td>3 flush</td>
<td>3 flush</td>
</tr>
<tr>
<td>Taps</td>
<td>13 min/8 l</td>
<td>9.3 min</td>
<td>13.6 min</td>
<td>9.3 min</td>
<td>13.6 min</td>
<td>9.3 min</td>
<td>13.6 min</td>
</tr>
<tr>
<td>Dishwasher</td>
<td>16 l</td>
<td>16 l</td>
<td>16 l</td>
<td>16 l</td>
<td></td>
<td>16 l</td>
<td></td>
</tr>
<tr>
<td>Clothes Washer</td>
<td>60 l</td>
<td>60 l</td>
<td>60 l</td>
<td>60 l</td>
<td></td>
<td>60 l</td>
<td></td>
</tr>
</tbody>
</table>
```
Exemplar #6 – Score: 2

- The work demonstrates a basic understanding of the situation.
  - tries to answer the question without any explicit mathematical calculations
- The strategy is unclear and/or incomplete.
  - while usage is given for each utility, there are no calculations
  - the water usage is inferred in the work, but the actual usage is never calculated or given
- The logic correctly references some aspects of the problem.
  - each utility is addressed, but without water usage given
- The reasoning is unclear.
  - the solution has some structure even though supporting calculations are not represented
  - while there is some attempt to logically address the problem, the end usage is not given

Show your work and write your final answer in the space provided.
Montrez votre travail et écrivez votre réponse finale dans l’espace prévu.

<table>
<thead>
<tr>
<th>Utility</th>
<th>Frequency</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shower</td>
<td>7 times a week</td>
<td>This comes well under the budget even we aren't</td>
</tr>
<tr>
<td>Toilet</td>
<td>25 times a week</td>
<td>using these resources too</td>
</tr>
<tr>
<td>Tap</td>
<td>40 times a week</td>
<td></td>
</tr>
<tr>
<td>Dishwasher</td>
<td>5 times a week</td>
<td>frugantly. You can take a shower a day and run</td>
</tr>
<tr>
<td>Clothes Washer</td>
<td>5 times a week</td>
<td>the dishwasher when full.</td>
</tr>
</tbody>
</table>
Exemplar #7 – Score: 1

- The work demonstrates an inadequate understanding of the situation.
  - addresses the concept of reduced water usage

- The strategy is unclear and/or incomplete.
  - while there is a limited attempt at reducing water usage, there are no calculations

- The logic is inadequate.
  - no mathematical calculations at all
  - only three utilities mentioned

- The reasoning is incomplete.
  - no calculations are shown
  - there is no solution

- Don't use bath(s) or shower at the same time
- Don't use dishwasher if you don't have much dishes to wash
- Use clothes washer only if you have the right amount of dirty laundry to wash.
Water Use

Exemplar #8 – Score: 1

- The work demonstrates an inadequate understanding of the situation.
  - there is no mention of reducing water usage
- The strategy is ineffective.
  - strategy is unclear
  - there are no calculations
- The logic does not reference the problem.
  - the bar graph is not labelled and there is no mention of the various utilities
- The reasoning is missing.
  - supporting calculations are not shown
  - there is no solution given
Exemplar #9 – Score: 0

☑️ The response does not address the purpose of the task.
● there is no explanation of the calculation, and no solution given

\[
11.15 \text{ min} \times 5 \times 7 = \frac{390.25 \text{ min}}{\text{week}}
\]

\[
\times 10 = \frac{3902.5 \text{ L}}{\text{week}}
\]

Btw my hand is broken so that is why my writing is bad.
Understanding Our Past: Pit Houses

Archaeologists study artifacts and monuments to understand past cultures. In the interior of British Columbia, First Peoples lived in circular homes, called pit houses. Pit houses varied in size, depending on how many people lived in the home.

Overhead Views

- Covered Structure
- Approximate Floor Area Required
  - Family
    - Area: 10 m²
  - Couple
    - Area: 7 m²
  - Adult
    - Area: 4 m²
  - Fire Pit and Ladder
    - Area: 1 m²

Fire pit 1 m²
13. Archaeologists examine the remains of a village of pit houses. Impressions of where the pit houses were located are still visible. The diagram below shows the remains of a village with a number of pit houses.

Estimate the number of people that could have lived in this village. State any assumptions made.

Explain and justify your solution.

This question is to be answered on paper.

**Summary of Requirements for a Level 4:**

- State initial assumptions
- Calculate area of small and large pit houses
- Estimate number of people
- Provide evidence to justify estimate
- Communicate solution in context
There are many possible solutions; several solutions are given.

**Solution 1**

Assumptions:

- 1 family consisting of 2 adults and 2 children requires 10 m² of living space
- ladder and fire pit require an area of 1 m² per pit house
- the same configuration of people live in each of the small and large pit houses
- all pit houses are occupied to their full capacity

There are 11 pit houses – 3 small with diameter of 7 m and 8 large with diameter of 11 m.

**Area of a small pit house:**

\[ A = \pi r^2, \quad r = \frac{d}{2} \]

\[ A = \pi (3.5)^2 \]

\[ A = 38.48 \text{ m}^2 \]

Area available for living:

\[ 38.48 \text{ m}^2 - 1 \text{ m}^2 \text{ (for fire pit and ladder)} = 37 \text{ m}^2 \]

**Number of people who lived in a small pit house:**

\[ \frac{37 \text{ m}^2}{10 \text{ m}^2/\text{family}} = 3 \text{ families} \]

Area required for 3 families: 3(10 m²) = 30 m²

37 m² – 30 m² = 7 m² left over

Area required for 1 couple = 7 m²

Total number of people who lived in a small pit house: 3(4) + 1(2) = 14

Since there are 3 small pit houses: 3(14) = 42

**Number of people who lived in a large pit house:**

\[ \frac{94 \text{ m}^2}{10 \text{ m}^2/\text{family}} = 9 \text{ families} \]

Area required for 9 families: 9(10 m²) = 90 m²

94 m² – 90 m² = 4 m² left over

Area required for 1 adult = 4 m²

Total number of people who lived in a large pit house: 9(4) + 1 = 37

Since there are 8 large pit houses: 8(37) = 296

Total number of people: 42 + 296 = 338

The estimated number of people who lived in the village is 338. This is reasonable under the above-mentioned assumptions. This solution also assumes that people are evenly distributed amongst the pit houses and that the maximum living space within each pit house is fully utilized.
Solution 2

Assumptions:

- 1 adult needs 4 m² of living space
- 1 couple needs 7 m² of living space
- 1 family consisting of 2 adults and 2 children requires 10 m² of living space
- ladder and fire pit require an area 1 m² per pit house
- the same configuration of people live in each of the small and large pit houses
- the maximum amount of living space in each pit house is used

There are 11 pit houses – 3 small with diameter of 7 m and 8 large with diameter of 11 m.

Area of a small pit house:
\[ A = \pi r^2, \quad r = \frac{d}{2} \]
\[ A = \pi (3.5)^2 \]
\[ A = 38.48 \text{ m}^2 \]
Area available for living:
\[ 38.48 \text{ m}^2 - 1 \text{ m}^2 (\text{for fire pit and ladder}) = 37 \text{ m}^2 \]

Strategy: Guess and check:

Number of people living in small pit houses:
Area required for 2 families: (2)(10 m²) = 20 m²
Area required for 4 adults: (4)(4 m²) = 16 m²
Area required is 20 m² + 16 m² = 36 m², which is less than 37 m² that is available.
Number of people: (2)(4) + 4 = 12 people per house
\[ \therefore (3)(12) = 36 \text{ people lived in the 3 small pit houses}. \]

Number of people living in large pit houses:
Area required for 6 families: (6)(10 m²) = 60 m²
Area required for 3 adults: (3)(4 m²) = 12 m²
Area required for 3 couples: (3)(7 m²) = 21 m²
Area required is 60 m² + 12 m² + 21 m² = 93 m², which is less than 94 m² that is available.
Number of people: (6)(4) + 3 + (3)(2) = 33 people per house
\[ \therefore (8)(33) = 264 \text{ people lived in the 8 large pit houses}. \]

36 + 264 = 300 people

The estimated number of people living in the village is 300. This is a reasonable estimate under the above-mentioned assumptions. However, some families may have more children than 2, thus requiring more space. On the other hand, some families may have had only one child thus requiring less space.
Solution 3

Assumptions:

- 1 adult needs 4 m² of living space
- 1 couple needs 7 m² of living space
- 1 family consisting of two adults and two children requires 10 m² of living space
- ladder and fire pit require an area of 1 m² per pit house
- different configurations of people live in each of the small pit houses and a few of the large pit houses

There are 11 pit houses – 3 small with diameter of 7 m and 8 large with diameter of 11 m.

**Number of people living in the 3 small pit houses:**

Pit House 1:
- Area required for 2 families + 4 adults: \( (2)(10 \text{ m}^2) + (4)(4 \text{ m}^2) = 36 \text{ m}^2 \)
- \( (2)(4) + 4 = 12 \text{ people} \)

Pit House 2:
- Area required for 5 couples: \( (5)(7 \text{ m}^2) = 35 \text{ m}^2 \)
- \( (5)(2) = 10 \text{ people} \)

Pit House 3:
- Area required for 1 family + 2 couples + 3 adults: \( (1)(10 \text{ m}^2) + (2)(7 \text{ m}^2) + (3)(4 \text{ m}^2) = 36 \text{ m}^2 \)
- \( (1)(4) + (2)(2) + 3 = 11 \text{ people} \)

Total number of people who lived in the small pit houses: \( 12 + 10 + 11 = 33 \text{ people} \).

**Strategy: Guess and Check**

**Number of people living in the 11 large pit houses:**

5 large pit houses occupied by families only – 9 families in each house
- Area required for 9 families: \( (9)(10 \text{ m}^2) = 90 \text{ m}^2 \)
- \( (9)(4) = 36 \text{ people in one large pit house} \)
- \( (36)(5) = 180 \text{ people in 5 large pit houses} \)

3 large pit houses occupied by 11 adults and 7 couples:
- Area required for 11 adults and 7 couples: \( (11)(4 \text{ m}^2) + (7)(7 \text{ m}^2) = 44 \text{ m}^2 + 49 \text{ m}^2 = 93 \text{ m}^2 \)
- \( 11 + (7)(2) = 25 \text{ people} \)
- \( (25)(3) = 75 \text{ people} \)

Total number of people who lived in the large pit houses: \( 180 + 75 = 255 \text{ people} \)
- \( 33 + 255 = 288 \text{ people} \)

The estimated number of people living in the village is 288. This is a reasonable estimate under the above-mentioned assumptions. However, some families may have more children than 2, thus requiring more space. On the other hand, some families may have had only one child, thus requiring less space.
Pit Houses

Solution 4

Assumptions

- 1 adult needs 4 m² of living space
- 1 couple needs 7 m² of living space
- 1 family consisting of 2 adults and 2 children requires 10 m² of living space
- ladder and fire pit require an area of 1 m² per pit house
- considering the total area of living space across all pit houses rather than by individual pit house

There are 11 pit houses – 3 small with diameter of 7 m and 8 large ones with diameter of 11 m.

Area of a small pit house:  
\[ A = \pi r^2, \quad r = \frac{d}{2} \]
\[ A = \pi (3.5)^2 \]
\[ A \approx 38.48 \text{ m}^2 \]

Area available for living:  
\[ 38.48 \text{ m}^2 - 1 \text{ m}^2 (\text{for fire pit and ladder}) = 37 \text{ m}^2 \]

Area of a large pit house:  
\[ A = \pi r^2, \quad r = \frac{d}{2} \]
\[ A = \pi (5.5)^2 \]
\[ A \approx 95.03 \text{ m}^2 \]

Area available for living:  
\[ 95.03 \text{ m}^2 - 1 \text{ m}^2 (\text{for fire pit and ladder}) = 94 \text{ m}^2 \]

Total area available for living:  
\[ (3)(37 \text{ m}^2) + (8)(94 \text{ m}^2) = 863 \text{ m}^2 \]

Number of people living in the pit houses:

\[ \frac{863 \text{ m}^2}{10 \text{ m}^2/\text{family}} = 86 \text{ families} \]

∴ 86 families can live in this area

Area required for 86 families:  
\[ 86(10 \text{ m}^2) = 860 \text{ m}^2 \]
\[ 863 \text{ m}^2 - 860 \text{ m}^2 = 3 \text{ m}^2 \text{ left over} \]

3 m² which is not enough room for a single adult or couple to live

Total number of people:  
\[ 86(4) = 344 \text{ people} \]

Therefore, my estimate would be 344 people lived in the village.

The estimate of 344 people is reasonable under the above-mentioned assumptions. However it is also likely that there were couples and single adults living in this village. Considering the total area of living space across all pit houses rather than by individual pit house made my calculations faster with fewer steps. However, this assumption may not properly reflect the real-life scenario if the houses have a considerable amount of space unoccupied.
Exemplar #1 – Score: 4

- The work demonstrates a proficient understanding of the situation.
  - combined the area requirements to efficiently fill available space in each pit house
  - assumptions are evident: the area combinations reflect the context of a village – population consists of families, couples, and adults

- The strategy is effective and comprehensive.
  - all area requirements are reflected in the solution
  - the total population is based on the distributions of people in each size of pit house

- The logic references all aspects of the problem.
  - determined the available floor area for each size of pit house \[\text{area} = \pi \text{(radius)}^2\]
  - used the area requirements to determine the number of people that could occupy each size of pit house
  - provided a total population for the village based on the number of each size of pit house

- The reasoning is clear, detailed, and organized.
  - areas allocations are shown for each size of pit house
  - distributions of people are shown for each size of pit house
  - total area and total population calculations are evident

---

**Assumptions:**
- Families need 10m² living space.
- Couples need 7m² living space.
- Adults need 4m² living space.
- There will be a combination of families, couples and adults in the community.
- Fire/ladder requires 1m².

**Small pit house:**

\[A = \pi r^2\]

- Area = \(\pi (3.5)^2\)
- Area = 38.48 m²

\[28 m^2 \div 1 m^2 \text{ per family} = 28 \text{ m² living space per family}\]

- Familys will be the most efficient use of space, so 37 m² = 3 families, plus 7 m² left of space.

- 7 m² is enough room for a couple.

So a small pit house can fit 3 families and couple, which is 14 people.

There are 3 small pit houses, 3 x 14 = 42 people.

---

**Large pit house area:**

\[A = \pi r^2\]

- Area = \(\pi (5.5)^2\)
- Area = 95.04 m²

- 95 m² / 1 m² per pit = 95 m²

- If we assume that about half of the pit houses are for families and the other half are for adults and couples, then we can assume about 5 families live in the pit house. totaling 50 m² living space.

- 44 m² = 44 m² left over.

- If 5 couples = 14 m² space = 35 m².

- 44 m² = 35 m² = 9 m² left which is enough space for 8 adults, with 1 m² left.

- Therefore, 1 large pit house can hold 5 families + 5 couples + 2 adults

- 7 \(\times\) 5 \(\times\) 2 = 70 people

- Small pit houses x 32 people = 94 people

- Large pit houses + large pit houses:

- \(42 + 38\) \(\times\) \(10\) = \(800\) people

- Estimated: about 339 people live in the village, using the stated assumptions.
Exemplar #2 – Score: 4

☑️ The work demonstrates a proficient understanding of the situation.
  • combined various area requirements to efficiently fill available space in each pit house
  • assumptions are evident: the area combinations reflect the context of a village –
    population consists of families, couples, and adults

☑️ The strategy is effective and comprehensive.
  • all area requirements are reflected in the solution
  • the total population is based on the distributions of people in each size of pit house

☑️ The logic references all aspects of the problem.
  • determined the available floor area for each size of pit house \([\text{area} = \pi (\text{radius})^2]\)
  • used the area requirements to determine the number of people that could occupy each
    size of pit house
  • provided a total population for the village based on the number of each size of pit house

☑️ The reasoning is clear, detailed, and organized.
  • areas allocations are shown for each size of pit house
  • distributions of people are shown for each size of pit house
  • total area and total population calculations are evident

\[\text{Assumptions:}\]
- Families are the most likely to live in the community
- Most pit houses will have families
- Each family requires 10m² living space
- Each pit house has a 1m² fire pit

\[\text{Small pit house:}\]
\[A = \pi (1)^2 = 3.1416 \text{m}^2\]
\[38.4 \times 3 \text{ houses} = 115.2 \text{m}^2 \times \left(10 \text{m}^2 \right) = 1,152 \text{m}^2\]
\[115.2 \text{m}^2 = 8 \text{ families with } 3.1416 \text{m}^2 \text{ left over}\]
\[8 \text{ m}^2 = 114 \text{ people} = 1 \text{ extra family}\]

\[\text{Large pit house:}\]
\[A = \pi (2)^2 = 12.566 \text{m}^2\]
\[95.038 \times 8 \text{ houses} = 760.304 \text{m}^2\]
\[740 \text{ m}^2 \times 8 \text{ families} = 5,920 \text{m}^2\]
\[752 \text{ m}^2 = 75 \text{ families}\]
\[752 \times 4 \text{ people} = 300 \text{ people in large house}\]

\[300 \times 0.44 = 132 \text{ people}\]


I think there will be 344 people in the village. This is an estimate because combinations of rooms are not exactly the same. Therefore, if there were 344 people in the village, there would probably be

\[\text{single individuals and children in the village, and they take more space, so the final number probably is less than 344.}\]
Exemplar #3 – Score: 3

☑️ The work demonstrates an adequate understanding of the situation.
- used one of the area requirements to fill available space in each pit house
- assumptions were implied, but not clearly stated: the area combinations reflect adults only (no couples or children in the village)

☑️ The strategy is sensible but has some inconsistencies.
- only one area requirement is reflected in the solution
- the total population is based on the distributions of adults only in each size of pit house

☑️ The logic references all aspects of the problem.
- determined the available floor area for each size of pit house \( [\text{area} = \pi (\text{radius})^2] \)
- used the area requirements to determine the number of adults that could occupy each size of pit house; fire pit requirements were not included; couples and children were not included
- determined a total number of adults for the village based on the number of each size of pit house

☑️ The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
- only one area allocation is shown for each size of pit house
- distributions of people are shown for each size of pit house (but did not change allocations to make efficient use of different areas)
- total area and total population calculations are evident
Exemplar #4 – Score: 3

The work demonstrates an adequate understanding of the situation.
- used one of the area requirements to fill available space in each pit house
- assumptions were implied, but not clearly stated: the area combinations reflect each group, but not a combination of people

The strategy is sensible but has some inconsistencies.
- only one area requirement is reflected in the solution
- the total population is based on the distributions of adults OR families OR couples – not a combination

The logic references all aspects of the problem.
- determined the available floor area for each size of pit house \[\text{area} = \pi (\text{radius})^2\]
- used the area requirements to determine the number of people that could occupy the total area of all houses combined; fire pit requirements were not included
- determined a total number of people for the village based on the total area of all houses

The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
- only one area allocation is shown for each size of pit house
- distributions of people are shown for the total area of all houses (but did not consider allocations to make efficient use of different areas)
- total area and total population calculations are evident
Exemplar #5 – Score: 2

☑️ The work demonstrates a basic understanding of the situation.
  • used one of the area requirements area requirements to fill available space in each pit house
  • incorrect assumptions: the area combinations reflect adults only (no couples or children in the village)

☑️ The strategy is unclear and/or incomplete.
  • only one area requirement is reflected in the solution
  • the total population is based on the distributions of adults only in the total area of all houses

☑️ The logic correctly references some aspects of the problem.
  • incorrectly determined the available floor area for each size of pit house (used the diameter instead of the radius)
  • used the area requirements to determine the number of adults that could occupy each size of pit house; fire pit requirements were not included; couples and children were not included
  • determined a total number of adults for the village based on the number of each size of pit house

☑️ The reasoning is unclear; supporting calculations are not well represented; the solution has some structure.
  • only one allocation is shown for the total area of the all houses combined
  • distributions of people were the same for each size of pit house (no consideration made for efficient use of different areas)
  • total area and total population calculations are evident
Exemplar #6 – Score: 1

The work demonstrates an inadequate understanding of the situation.
• the concept of area was not applied to the situation
• assumptions not stated: unclear whether adults, couples, or children were considered to be in the village

The strategy is ineffective.
• diameters referenced instead of areas
• the total population is based on incorrect distributions of people in each size of pit house

The logic does not reference the problem. The solution may contain fundamental mathematical errors.
• the logic behind the values used towards a solution is not shown (allocations of areas were not considered for different combinations of people or for a fire pit)

The reasoning is missing or irrelevant; supporting calculations are not shown.

\[
\begin{align*}
11 & = \text{houses} \\
8 & = 11 \text{ m} \\
3 & = 7 \text{ m} \\
\text{If the 11 m house can fit approx.} & \\
4 & = \text{people} \quad \text{and the 7 m house can fit approx} \\
3 & = \text{people} \quad 4.8 \\
& = 63 \text{ people (estimate)} \\
3 & = 7 \text{ m. Can fit in this village}
\end{align*}
\]
Exemplar #7 – Score: 1

- The work demonstrates an inadequate understanding of the situation.
  - the concept of area was not applied to the houses
  - assumptions not stated: unclear whether couples and families were referenced but not linked to the logic of the problem

- The strategy is ineffective.
  - diameters are stated as areas
  - the total population is based on incorrect distributions of people in each size of pit house

- The logic does not reference the problem. The solution may contain fundamental mathematical errors.
  - the logic behind the values used towards a solution is not shown (allocations of areas for different combinations of people were unclear; area for the fire pit was not considered)

- The reasoning is missing or irrelevant; supporting calculations are not shown.

```
there are 8 11m² houses and 3 7m² houses

if a couple approximately needed 7m² that's

6 people if a family of 5 needed

space of 10m² that's 40 people 40 + 6 =

46 people live in the village
```
Exemplar #8 – Score: 1

- The work demonstrates an inadequate understanding of the situation.
  - the concept of area was referenced but incorrectly applied to the situation
  - assumptions not stated: unclear whether adults, couples, or children were considered to be in the village

- The strategy is ineffective.
  - diameters referenced instead of areas
  - the total population is based on incorrect area requirements

- The logic does not reference the problem. The solution may contain fundamental mathematical errors.
  - the logic behind the values used towards a solution is not shown (allocations of areas were not considered for different combinations of people or for a fire pit)
  - final solution does not reference a sensible value for populations (0.5 of a person)

- The reasoning is missing or irrelevant; supporting calculations are not shown.
Exemplar #9 – Score: 0

- The work is not relevant to solving the problem.
- Response does not address the purpose of the task.
  - area is not referenced
  - a value is stated, but is not supported or linked to the context of the problem
- The solution is incorrect with no work shown.
- Information is simply recopied from the problem. A population value is stated with no reference or support.

27. In the village, I estimate that there would be 139 people living in the pit house village.
Five years ago, Jae Eun and Ted, started a company together. They each contributed a different amount of money to start the company.

The Company created several popular video games. Now Jae Eun and Ted receive an offer from an individual who wants to buy their company.
26. Five years after the start of The Company, The Founders decide to sell it for $750 000. To determine their fair share of the sale price, they agree that any contributions made towards start-up costs will be worth 1.5 times their original value. Contributions made after start-up will not be adjusted.

How much should Jae Eun and Ted each receive from the sale of their company?

Explain and justify your solution.

This question is to be answered on paper.

Summary of Requirements for a Level 4:

- Determine the current value of the initial investments both partners made (Jae Eun $87 750 and Ted $33 000)
- Calculate the total investment over 5 years (Ted $108 000 and Jae Eun $87 750)
- Select and communicate a strategy for sharing the $750 000
- Provide evidence to justify solution
- Communicate solution in context

There are several possible solutions; a few solutions are given.

Solution 1

This solution is based on each partner receiving a percentage of the total sale price based on what they invested in the company.

The initial investment is worth 1.5 times its original value. Jae Eun and Ted contributed different amounts at the start.

Jae Eun: $50 000 + $8 500 = $58 500

$58 500 × 1.5 = $87 750 (Jae Eun’s initial contribution weighted at 1.5x)

Ted: $22 000 × 1.5 = $33 000 (Ted’s initial contribution weighted at 1.5x)

Ted continued to contribute $1250/month for 5 years.

$1250 × 5 years × 12 months/year = $75 000

In total, Ted invested: $33 000 + $75 000 = $108 000

Altogether, Jae Eun and Ted contributed: $87 750 + $108 000 = $195 750
The percentage of $195,750 that each partner contributed:

\[
\begin{align*}
\text{Jae Eun: } & \frac{87,750}{195,750} \times 100 = 44.8\% \\
\text{Ted: } & \frac{108,000}{195,750} \times 100 = 55.2\%
\end{align*}
\]

Therefore, Jae Eun should receive 45% (or 44.8%) of the sale price, while Ted should receive 55% (or 55.2%).

Jae Eun: $750,000 \times 0.45 = $337,500 (or $336,000 using decimal %)
Ted: $750,000 \times 0.55 = $412,500 (or $414,000 using decimal %)

Solution 2

This solution is based on splitting the profit of the sale after deducting the partners’ contributions.

The same initial calculations as in Solution 1:

Jae Eun: $50,000 + $8,500 = $58,500
\[58,500 \times 1.5 = 87,750 \text{ (Jae Eun’s initial contribution weighted at 1.5x)}\]
Ted: $22,000 \times 1.5 = $33,000 (Ted’s initial contribution weighted at 1.5x)

Ted continued to contribute $1250/month for 5 years.

$1250 \times 5 \text{ years} \times 12 \text{ months/year} = $75,000

In total, Ted invested: $33,000 + $75,000 = $108,000

Total Partner contributions: $87,750 + $108,000 = $195,750

Sale price of Company less Total Partner contributions: $750,000 – $195,750 = $554,250

Profit split equally: $554,250 \div 2 = $277,125 for each partner

Therefore:
Jae Eun: $87,750 + $277,125 = $364,875
Ted: $108,000 + $277,125 = $385,125
Solution 3

This solution is based on splitting the profit of the sale and crediting the difference between investment contributions to the partner that paid more.

The same initial calculations as in Solution 1:

Jae Eun: $50 000 + $8 500 = $58 500
    $58 500 \times 1.5 = $87 750 (Jae Eun’s initial contribution weighted at 1.5×)

Ted: $22 000 \times 1.5 = $33 000 (Ted’s initial contribution weighted at 1.5×)

Ted continued to contribute $1250/month for 5 years.

$1250 \times 5 \text{ years} \times 12 \text{ months/year} = $75 000

In total, Ted invested: $33 000 + $75 000 = $108 000

Ted invested more than Jae Eun: $108 000 – $87 750 = $20 250. 
Ted paid $20 250 more than Jae Eun.

Split the sale income: $750 000 ÷ 2 = $375 000

As Ted invested more in the company, he will receive $20 250 more than half the sale price, while Jae Eun will receive $20 250 less than half the sale price:

Jae Eun: $375 000 – $20 250 = $354 750
Ted: $375 000 + $20 250 = $395 250
Exemplar #1 – Score: 4

- The work demonstrates a proficient understanding of the situation.
  - determines the total investment over 5 years for each partner
  - allocates a percentage of the sale price to each partner based on their individual investment

- The strategy is effective and comprehensive.
  - percentage of sale price allocated is based on each partner’s total investment over 5 years

- The logic references all aspects of the problem.
  - each partner’s investment is considered when determining the allocation of sale price

- The reasoning is clear, detailed, and organized.
  - the calculations for investment and percentage contribution are included and correct
  - the allocation of sale price is determined and a rationale is provided

---

Joe: $35,500 x 1.5 = $53,250
Ted: $22,000 x 1.5 = $33,000 + $1750 x 12 months x 5 years = $7500
Total = $104,000

Joe’s percentage contribution: $53,250 \div 104,000 = 41.8%
Ted’s percentage contribution: $33,000 \div 104,000 = 31.7%

Joe should receive 41.8% of the money from the sale. Because that is the percent he contributed to the company in funds. This means that Joe should receive $336,000.
Ted should receive 31.7% of the money from the sale. The reason for this is because he contributed 41.8% to the company when it comes to money, so in theory he owns 31.7% of the company, so he should receive $114,000 of the $750,000.
Exemplar #2 – Score: 4

- The work demonstrates a proficient understanding of the situation.
  - determines the total investment over 5 years for each partner
  - determines that each partner should be allocated back their individual investments and then half of what remains from the sale price

- The strategy is effective and comprehensive.
  - from the sale price, each partner receives back their total investment over 5 years and then half of what remains

- The logic references all aspects of the problem.
  - each partner’s investment is considered when determining the allocation of sale price

- The reasoning is clear, detailed, and organized.
  - the calculations for investment are included and correct
  - the allocation of sale price is determined and is reasonable

---

Ted should receive $385,125.

Jae should receive $364,875.

I think they should each be paid back what they contributed.

Ted and Jae, and then have the rest of the money split in half.
Exemplar #3 – Score: 4

The work demonstrates a proficient understanding of the situation.
• determines the total investment over 5 years for each partner
• allocates a percentage of the sale price to each partner based on their individual investment

The strategy is effective and comprehensive.
• percentage of sale price allocated is based on each partner’s total investment over 5 years

The logic references all aspects of the problem.
• each partner’s investment is considered when determining the allocation of sale price

The reasoning is clear, detailed, and organized.
• the calculations for investment and percentage contribution are included and correct
• the allocation of sale price is determined and supported by sound mathematical reasoning

\[
\text{Joe contribution} = 58,500 \times 1.5 = 87,750\$ \\
\text{Ted Contribution} = 22,000 \times 1.5 + 60 \times 1250 = 195,750 \\
33,000 + 75,000 = 108,000\$ \\
108,000 \times 87,750 \\
\text{Ted contributes 55%} \\
\text{Jae contribute 45%} \\
108,000 \times 87,750 \\
\text{Ted gets 412,500\$} \\
\text{Jae gets 337,500\$} \\
\text{because of percentage of previous contribution.}
\]
Exemplar #4 – Score: 3

- The work demonstrates an adequate understanding of the situation.
  - determines the total investment over 5 years for each partner
  - allocates a percentage of the sale price to each partner based on their individual investment

- The strategy is sensible but has some inconsistencies.
  - percentage of sale price allocated is based on each partner’s total investment over 5 years

- The logic references all aspects of the problem.
  - each partner’s investment is considered when determining the allocation of sale price

- The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
  - the calculations for investment are included, but contain an error
  - the allocation of sale price is determined

Ted and Jae should both get $258,375 from selling the company as well as $87,760 for Jae’s contributions and $145,500 for Ted’s contributions.
Exemplar #5 – Score: 3

☑ The work demonstrates an adequate understanding of the situation.
  • determines the total investment over 5 years for each partner; however, start-up cost adjustment is not applied to one partner’s contribution
  • allocates a percentage of the sale price to each partner based on their individual investment

☑ The strategy is sensible but has some inconsistencies.
  • percentage of sale price allocated is based on each partner’s total investment over 5 years

☑ The logic references all aspects of the problem.
  • each partner’s investment is considered when determining the allocation of sale price

☑ The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
  • the calculations for investment are included, but contain an error
  • the allocation of sale price is determined and a rationale is provided

Sold for $750,000 after 5 years
Jae Eun - 58,500 x 1.5 = 87,750 (50,000 + 8,500)
Ted - 97,000 x 1.22 = 115,340 (1250 x 12 x 5)
58,500 + 97,000 = 184,750 - money put in (total)
Jae Eun = 184,750 / 3 = 61,583
Ted - 184,750 / 3 = 61,583
Their shares of the company
Jae = 750,000 x 0.47 = 352,500
Ted = 750,000 x 0.52 = 390,000

Jae Eun should receive $352,500 of the profit because she contributed 0.47 of the money to the company. Ted should receive $390,000 of the profit because he contributed 0.52 of the money to the company.
Exemplar #6 – Score: 2

The work demonstrates a basic understanding of the situation.
- determines the total investment over 5 years for each partner; however, calculations contain an error
- a portion of the sale price is allocated to each partner; however, the rationale for the allocation is unclear

The strategy is unclear and/or incomplete.
- although a portion of the sale price is allocated to each partner, it is unclear how that amount was determined

The logic correctly references some aspects of the problem.
- total investment of each partner is determined and each partner is allocated a portion of the sale price
- each partner’s investment has some bearing on the sale price allocation; however, a significant proportion of the sale price is not allocated

The reasoning is unclear; supporting calculations are not well represented; the solution has some structure.
- the calculations for investments are shown, but contain an error
- the calculations to determine sale price allocation are shown; however, rationale is not provided and amounts are unreasonable
Exemplar #7 – Score: 2

☑ The work demonstrates a basic understanding of the situation.
  • states the total investment over 5 years for each partner; however, supporting calculations are missing
  • a percentage of the sale price is allocated to each partner; however, there is no rationale to the allocation

☑ The strategy is unclear and/or incomplete.
  • although a percentage of the sale price is allocated to each partner, it is unclear how that percentage was determined

☑ The logic correctly references some aspects of the problem.
  • total investment of each partner is stated and each partner is allocated a percentage of the sale price
  • each partner’s investment has no bearing on the sale price allocation

☑ The reasoning is unclear; supporting calculations are not well represented; the solution has some structure.
  • the calculations for investment are not shown
  • no rationale is provided for how percentage allocation was determined
Video Game Company

Exemplar #8 – Score: 1

- The work demonstrates an inadequate understanding of the situation.
  - each partner’s investment is not addressed
  - half of the sale price is allocated to each partner; however, there is no rationale to the allocation

- The strategy is ineffective.
  - although half of the sale price is allocated to each partner, it is not evident how that portion was determined

- The logic does not reference the problem. The solution may contain fundamental mathematical errors.
  - each partner’s investment has no bearing on the sale price allocation

- The reasoning is missing or irrelevant; supporting calculations are not shown.

\[
\frac{750,000}{2} = 375,000
\]

They should get 375,000 dollars each.
Exemplar #9 – Score: 1

- The work demonstrates an inadequate understanding of the situation.
  - start-up cost adjustment is applied to each partner’s initial investment; however, total investment over 5 years is not calculated
  - a portion of the sale price is allocated to each partner; however, there is no rationale to the allocation
- The strategy is ineffective.
  - although a portion of the sale price is allocated to each partner, it is incorrect given the context of the problem
  - a significant proportion of the sale price is not allocated, with no rationale provided
- The logic does not reference the problem. The solution may contain fundamental mathematical errors.
  - although each partner’s initial investment has some bearing on the sale price allocation, the allocation is illogical
- The reasoning is missing or irrelevant; supporting calculations are not shown.

\[
\begin{align*}
22,000 \times 1.5 &= 33,000 \\
58,500 \times 1.5 &= 87,750 \\
\frac{120,750}{?} &= \\
750,000 &= 10,000,000 \\
They \ will \ each \ receive \$120,750.
\end{align*}
\]
Exemplar #10 – Score: 0

☑ An incorrect mathematical solution with no work shown.

Ted: $450,000

Jae Eun: $300,000
You are being trained in managing forest fires. Your Fire Fighting Training Manual describes a forest fire spread simulation.

The map below is a fire grid. It describes the likelihood of a fire spreading to different cells.

<table>
<thead>
<tr>
<th>Fire-Spread Rating</th>
<th>Probability of Spreading</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Fire has a 0% chance of spreading into it from neighbouring cells</td>
</tr>
<tr>
<td>1</td>
<td>Fire has a 50% chance, or probability of 0.5, of spreading into it from neighbouring cells</td>
</tr>
<tr>
<td>2</td>
<td>Fire has an 80% chance, or probability of 0.8, of spreading into it from neighbouring cells</td>
</tr>
</tbody>
</table>
Example: At time zero, the forest in the highlighted cell (🔥) is burning. There are many possible ways that the fire can spread. Two possible scenarios are shown below.

**Scenario 1**
3 neighbouring cells are burning.

**Scenario 2**
5 neighbouring cells are burning.

5 more neighbouring cells are burning.

Strong wind conditions can affect the spread of fire into neighbouring cells.

Wind from the south-east

Wind from the west

Wind increases the probability of fire spreading into neighbouring cells which are downwind, and wind decreases the probability of fire spreading into neighbouring cells which are upwind:

<table>
<thead>
<tr>
<th>Fire-Spread Rating</th>
<th>Probability of Fire-Spreading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Neighbouring (no wind)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
</tr>
</tbody>
</table>
26. At time zero, the highlighted cell is burning and there is a strong, constant wind blowing from the south. There are people living in the areas within cell A and cell B.

What is the minimum time it would take the fire to reach cells A and B? What is the likelihood of the fire spreading to cells A and B within that time?

Explain and justify your solution.

Summary of Requirements for a Level 4:
- Provide time for fire to get to cell A and cell B (minimum 8 hours for both)
- Likelihood of the fire getting to cell A within that time (within 8 hours, it is high)
- Likelihood of the fire getting to cell B within that time (within 8 hours, it is very low)
- Provide evidence to justify solution
- Communicate solution in context
There are several possible solutions; a couple of solutions are given.

**Solution 1**

Cell A: The soonest the fire could reach cell A is 8 hours. As cell A is downwind from the highlighted cell, there is a high probability that the fire will spread to that cell in 8 hours (64%).

\[ \text{Probability} = 1.0 \times 0.8 \times 0.8 \times 1.0 = 0.64 \]

Cell B: The soonest the fire could reach cell B is 8 hours, but because it is upwind, there is a low probability (0.24%) that the fire will spread to that cell.

\[ \text{Probability} = 0.2 \times 0.2 \times 0.3 \times 0.2 = 0.0024 \]

**Solution 2**

The fastest the fire could reach both indicated cells is 8 hours. However, the fire is more likely to spread to cell A as it is downwind, and the most probable path is indicated by the grid below. The fire is not likely to spread to cell B because it is upwind. The most probable path is indicated by the grid below.
Note:
There are 6 ways the fire could reach cell B in 8 hours, and the actual probability of spreading to cell B in that time is the sum of all of those cases \( P = 0.028 \). It is sufficient for students to provide one case and give that probability.

\[
(0.2)^3(0.3) = 0.0024
\]

\[
(0.2)^4 = 0.0016
\]

\[
(0.2)^3(0.3) = 0.0024
\]

\[
(0.2)^4 = 0.0016
\]

\[
(1.00)(0.2)^3 = 0.008
\]

\[
(1.00)(0.2)^2 (0.3) = 0.012
\]
Exemplar #1 – Score: 4

The work demonstrates a proficient understanding of the situation.
- describes a likely path the fire could follow while spreading to both target cells
- identifies the correct probabilities based on the fire spread rating and the wind direction

The strategy is effective and comprehensive.
- the probabilities for both (complete) paths are calculated
- total times for both paths are determined

The logic references all aspects of the problem.
- overall probability is the product of the probabilities for each stage
- the total times are based on the number of stages within each path

The reasoning is clear, detailed, and organized.
- the calculations for total time are described
- the probabilities for each stage are shown and correlate with the data table in the question
- the paths are drawn on a grid
Exemplar #2 – Score: 3

The work demonstrates an adequate understanding of the situation.
• work is based on a likely path the fire could follow while spreading to both target cells – but the paths are not described
• identifies the correct probabilities based on the fire spread rating and wind direction

The strategy is sensible but has some inconsistencies.
• the probabilities for both (complete) paths are calculated
• the total times are shown without support

The logic references all aspects of the problem.
• overall probability is the product of the probabilities for each stage
• time for the complete path is shown

The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
• the calculations for total time are not shown
• the probabilities for each stage are shown and correlate with the data table in the question
• the paths are not described or drawn on a grid
Exemplar #3 – Score: 3

The work demonstrates an adequate understanding of the situation.
- work is based on a likely path the fire could follow while spreading to both target cells
- identifies the correct probabilities based on the fire spread rating and wind direction

The strategy is sensible but has some inconsistencies.
- the probabilities for both (complete) paths are calculated
- the total times are shown without support

The logic references all aspects of the problem.
- overall probability is the product of the probabilities for each stage
- time for the complete path is shown

The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
- the calculations for total time are not shown
- the probabilities for each stage are shown and correlate with the data table in the question
- the paths are drawn on a grid

A. is upwind downwind
Fire could reach A in 8 hours
The probability is
\( (1.0)(0.8)(0.8)(1.0) = 0.64 \)

B. is downwind upwind
Fire could reach B in 8 hours
The probability is
\( (0.2)(0.2)(0.8)(0.2) = 0.0024 \)

The probability that the fire reaches A in 8 hours is 64%, which is fairly high.
The fire can reach B in 8 hours, but the probability is very low, 0.0024, which is not very likely.
Exemplar #4 – Score: 3

The work demonstrates an adequate understanding of the situation.
• work is based on a likely path the fire could follow while spreading to both target cells
• applies some of the correct probabilities based on the fire spread rating and wind direction

The strategy is sensible but has some inconsistencies.
• the overall probabilities are shown but not supported (one is incorrect)
• total times for both paths are determined

The logic references all aspects of the problem.
• overall probability is the product of the probabilities for each stage (one is incorrect)
• total times are based on the number of stages within each path

The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
• total time is determined based on the sequence of drawings
• calculation of the probabilities for each stage are not shown
• the paths are drawn on a grid

continued on next page
The fire will reach location A in 10 hours. The people should evacuate as the fire will arrive soon. The probability is 100%.

The fire will reach location B in 10 hours, however the probability is 0.24%. So the people are probably safe.
Exemplar #5 – Score: 2

- The work demonstrates a basic understanding of the situation.
  - work is based on a likely path the fire could follow— but just to one of the target cells
  - limited reference to the concept of probability

- The strategy is unclear and/or incomplete.
  - overall probabilities are not determined for either path
  - the total times are based on the number of stages, but for just one of the paths

- The logic correctly references some aspects of the problem.
  - neither the single cell or total probabilities are addressed
  - time for one complete path is presented; the other is not referenced

- The reasoning is unclear; supporting calculations are not well represented; the solution has some structure.
  - the total time is determined based on the sequence of drawings
  - the probability calculations are not shown
  - one of the paths is described using a sequence of grids
Exemplar #6 – Score: 2

- The work demonstrates a basic understanding of the situation.
  - work is based on a likely path the fire could follow— but just to one of the target cells
  - limited reference to the concept of probability

- The strategy is unclear and/or incomplete.
  - the overall probabilities are shown but not supported (both are incorrect)
  - the total times are inferred in the diagram, but the process is not clear

- The logic correctly references some aspects of the problem.
  - time for one complete path is inferred; the time for the other path is not referenced
  - the single cell probabilities are not shown or referenced

- The reasoning is unclear; supporting calculations are not well represented; the solution has some structure.
  - the total time is not supported
  - the probability calculations are not shown
  - the paths are inferred on a grid
Exemplar #7 – Score: 2

- The work demonstrates a basic understanding of the situation.
  - addresses the concept the total time for the fire to spread to the target cells
  - references the concept of probability

- The strategy is unclear and/or incomplete.
  - the probabilities are based on conjecture
  - there is some explanation of the total times involved for the fire to spread

- The logic correctly references some aspects of the problem.
  - fragmented logic determining a total time
  - the single cell probabilities are referenced but not connected

- The reasoning is unclear; supporting calculations are not well represented; solution has limited structure.
  - the probability calculations are not shown
  - the paths are described using grids; the total time is represented on the drawings
Exemplar #8 – Score: 1

- The work demonstrates an inadequate understanding of the situation.
  - the time for the fire to spread is stated (incorrectly) but not supported
  - probabilities are not considered
- The strategy is ineffective.
  - a strategy is not evident
- The logic does not reference the problem. The solution may contain fundamental mathematical errors.
  - the logic behind the solution is not shown
- The reasoning is missing or irrelevant; supporting calculations are not shown.

It would probably take around 3-4 hours to reach B with the wind coming from the west.

For A I think the least amount of time would be 4.5 hours.
Exemplar #9 – Score: 1

☑ The work demonstrates an inadequate understanding of the situation.
  • the time for the fire to spread is stated but not supported
  • probabilities are restated

☑ The strategy is ineffective.
  • the probabilities between single cells are restated, but the overall probability is not considered
  • the total times are stated without any supporting rationale

☑ The logic does not reference the problem. The solution may contain fundamental mathematical errors.
  • time for the complete path is presented without supporting logic
  • total probabilities are not addressed

☑ The reasoning is missing or irrelevant; supporting calculations are not shown.

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The work demonstrates an inadequate understanding of the situation.

The strategy is ineffective.

The logic does not reference the problem. The solution may contain fundamental mathematical errors.

The reasoning is missing or irrelevant; supporting calculations are not shown.
Exemplar #10 – Score: 1

- The work demonstrates an inadequate understanding of the situation.
  - the time for the fire to spread is stated but not supported
  - probabilities are restated

- The strategy is ineffective.
  - the probabilities between single cells are restated, but the overall probability is not calculated
  - the total times are stated without any supporting rationale

- The logic does not reference the problem. The solution may contain fundamental mathematical errors.
  - time for the complete path is presented without supporting logic
  - total probabilities calculations are incomplete

- The reasoning is missing or irrelevant; supporting calculations are not shown.

Minimum time fire to spread to A and B

A: The fire will reach point A in 5 hours

0.3 x 0.2 =

B: Point B is below the point where the fire starts and because of that and the upward wind, point B isn’t affected.
The work demonstrates an inadequate understanding of the situation.
- the time for the fire to spread is shown but is incorrect
- the concept of probabilities are discussed in vague terms

The strategy is ineffective.
- overall probability is presented but not supported
- the total times are based on the number of stages for each path

The logic does not reference the problem. The solution may contain fundamental mathematical errors.
- time for the complete path is presented
- total probabilities are shown but not calculated

The reasoning is missing or irrelevant; supporting calculations are not shown.
- the probabilities for each stage are shown and correlate with the data table in the question
- the paths are described; the total time is represented on the drawings
Exemplar #12 – Score: 0

- The work is not relevant to solving the problem.
- Response does not address the purpose of the task.
  - probability is not referenced
  - a time is stated, but is not supported or linked to the context of the problem
- The solution is incorrect with no work shown.
- Information is simply recopied from the problem. A time value is stated with no reference or support.

It would take about 5 hours