GRADE 8 MATHEMATICS: Using Math and Statistics to Estimate Volume of a Lake

<table>
<tr><th colspan="3">Summary of Learning Opportunity</th></tr>
<tr><td colspan="3">I believe that all students are capable of thinking mathematically and statistically. In my class, we focus on critical thinking in a numerate way; students who are still developing math facts and number sense are very capable of conceptualizing and strategizing at a very high level.
In this unit, students first learned a variety of mathematical and statistical concepts: area using formulas, area using grids, surface area and volume, random sampling, topography, and random sampling for area and volume. We discussed the advantages and limitations of each method. Students then estimated the area and volume of an irregularly shaped local lake by choosing one of the methods.</td></tr>
<tr><td>Curricular Competencies and Content</td><td>Mathematics 8</td><td>• Estimate reasonably
• Model mathematics in contextualized experiences
• Apply multiple strategies to solve problems in both abstract and contextualized situations
• Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving
• Represent mathematical ideas in concrete, pictorial, and symbolic forms
• Numerical proportional reasoning (rates, ratio, proportions, and percent)
• Surface area and volume of regular solids, including triangular and other right prisms and cylinders
• Construction, views, and nets of 3D objects
• Theoretical probability with two independent events</td></tr>
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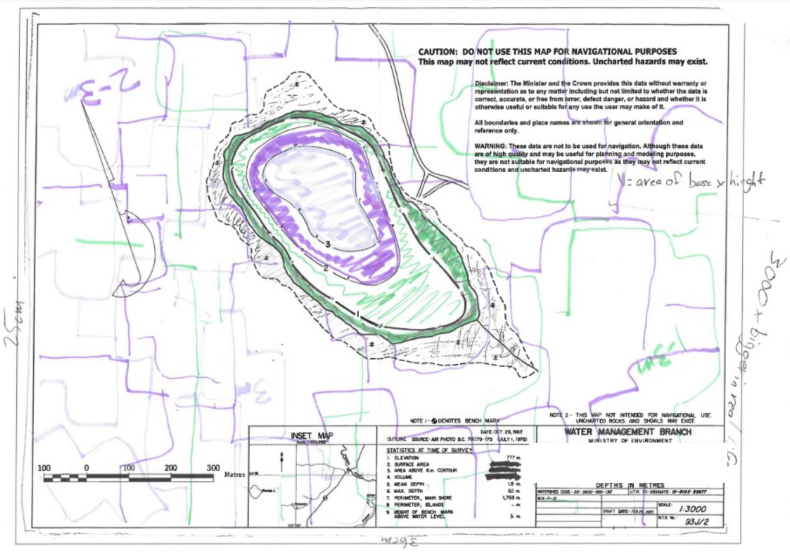
Numeracy Connections	Instruction and Assessment	Competencies Developed, Practiced, and/or Assessed
NUMERACY: Interprets—Understands the real-world problem ➡	Our class learned several math and stats models through a series of context-building activities in our unit. We discussed the advantages and limitations of using each model. • Area and volume using formulas: reviewed calculating area and volume of basic shapes • Area using grids: counting the number of full and partial squares to find the area of an irregular shape • Random sampling: We conducted a fun poll of the class (pancakes vs. waffles) and determined the proportion of votes for each choice. We then replicated the poll with random sampling, choosing tokens from a bucket (tokens were proportional to the initial poll, with a colour representing pancakes and another to represent waffles). As the number of trials increased (Law of Large Numbers), the ➡	• Apply multiple strategies to solve problems in both abstract and contextualized situations • Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving

ratio of randomly sampled token colours became closer to the results of the initial poll.

- Random sampling for area: Students first measured an irregular shape (image of an octopus) using the grid method. Then we used random sampling to determine the area by dropping paper hole punches onto the image and comparing the proportion of those that fell onto the image to the total number of pieces that fell outside the image. The random sampling proportion was equivalent to the proportion of the octopus' area to the total area of the paper.
- Topography: We viewed and discussed a topographical map, visualizing the landscape as slices. We practiced drawing 3D models of the hills on the map to help visualize.
- Random sampling for volume: We discussed that since volume can be described as the area of a shape's base multiplied by its height, we could approximate an object's volume as long as the object had a uniform cross section (layer or slice as with the topo maps); we could estimate the area using random sampling.

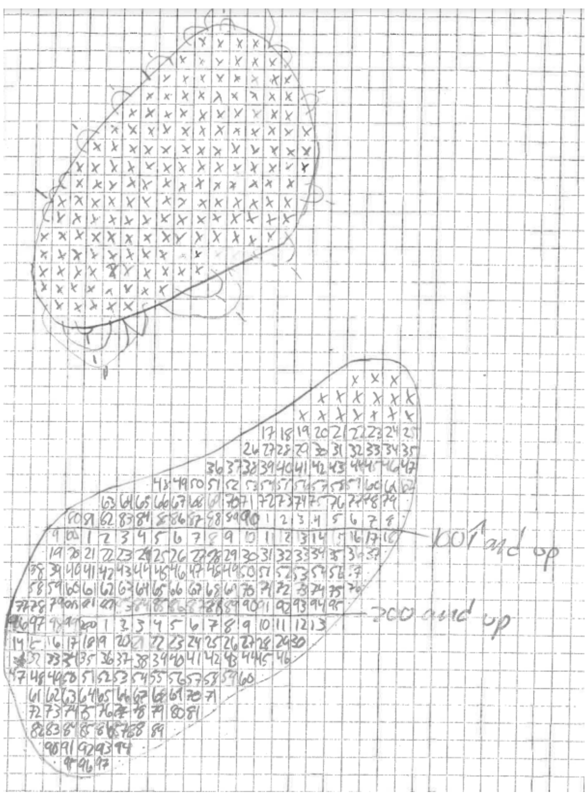
NUMERACY: Applies--Translates the scenario into a mathematical problem (mathematizes); Solves--Estimates reasonably in context; Solves-- Solves the mathematical problem →	2. For the summative task, students were given a bathymetric map of a local lake. We discussed how a bathymetric map was the same idea as a topographic map but for bodies of water. The students' task was to use any method of their choice (using familiar shapes, grids, random sampling, etc.) to determine the volume of the lake. →	• Estimate reasonably • Model mathematics in contextualized experiences • Apply multiple strategies to solve problems in both abstract and contextualized situations • Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving • Represent mathematical ideas in concrete, pictorial, and symbolic forms

Proficient Student Work, Teacher's Assessment and Reflection



Teacher's Observations and Assessment

This student is **proficient** in that they were able to use the methods we learned in a logical and organized way (calculating the area using a grid, interpreting the bathymetry as layers); along with their own method of tracing the layers and using the map scale to determine the actual dimensions of the lake. The student chose to use a smaller grid size as it was easier to estimate partial squares because they were smaller and would therefore be more accurate. The student made some errors in unit conversions and scaling when finding the total volume. Overall, this student was able to use the models learned in this unit together to solve the problem in a way that made sense to them, and demonstrated proficient thinking.



Layer 3

80.5 = layer 3

20.125 cm² x 3000²

181,125,000 cm² x 100

= 1.81125 x 10¹⁰

= 181,125,000,000 cm³ → Volume

Layer 2

43 cm² x 3000²

= 387,000,000 m² x 100

= 3.87 x 10¹⁰

= 38,700,000,000 cm³ → Volume

Layer 1

80.75 m² x 3000²

= 726,750,000 cm² x 100

= 7.2675 x 10¹⁰

= 72,675,000,000 cm³

Final Volume

V = L1 + L2 + L3

= 1.294875 x 10¹¹

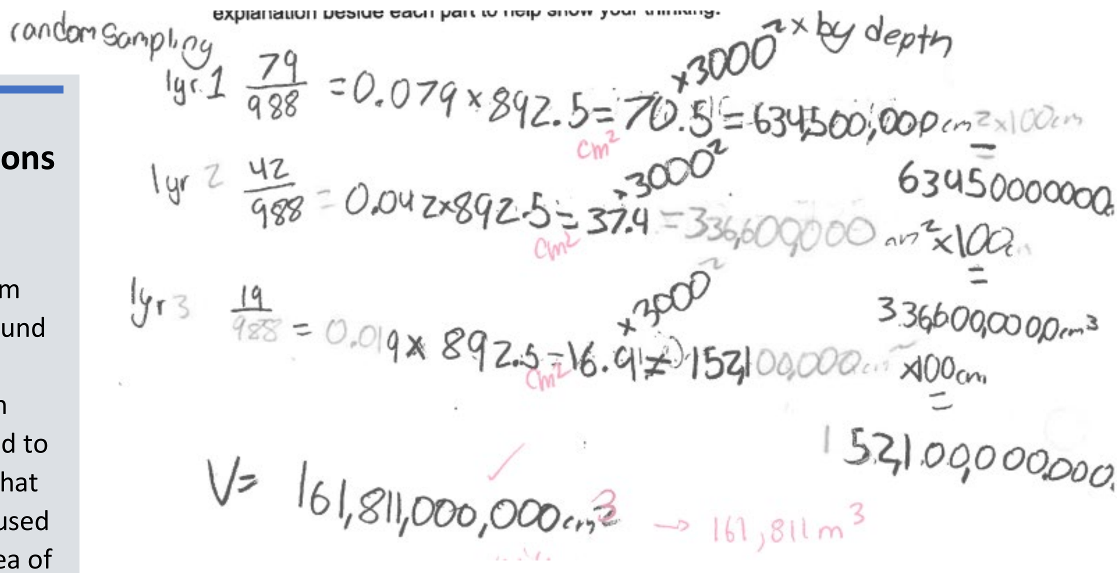
= 129,487,500,000 cm³

= 129,487.5 m³

Layer 1 (depth of 1m): 8 — 12.5 % or 1/8 of the map

Layer 2 (depth of 2m): 16 — 6.25 % or 1/16 of the map

Layer 3 (depth of 3m): 32 — 3.1% or 1/32 of the map



Teacher's Observations and Assessment

This student used a random sampling method. They found the ratio of randomly generated dots that fell on each depth layer compared to the total number of dots that landed on the map. They used these ratios to find the area of each layer using proportional relationships. For depth, the student estimated the relative size of each bathymetric slice. They did require some guidance when using scale and unit conversions.

The student took a logical approach to finding each layer's area and total volume. and was able to use multiple methods to either estimate or calculate the volume of the lake. They applied their understanding of proportional relationships, probability, area and volume to solve this task. This student is **proficient**.

Teacher's Reflection

The students were able to grasp the mathematical and statistical concepts quite well. I used the Numeracy aspect of Interpret during my project planning as a reminder to set the stage and build the context for my students.

When I spoke with students about random sampling, we had some great discussions about how the number of dots that landed in each depth layer compared to the total number of dots that fell on the map was proportional to that layer's area to the overall map area. Some students also did many more trials on their own to increase the sample size. The method we used to determine the volume ended up being around ⅔ of the actual volume as we visualized the lake in "layers" or "slices" which didn't end up being precise enough. This would have been an excellent discussion or extension to build student understanding of the limitations of a method. I would also like to find a more time efficient method for random sampling: we could pool classroom data to cut back on work load, increase sample size, and improve accuracy.

I would do this project again. Through observing students as they worked, I learned about their numerate thinking processes. For example, we had to have a discussion about how squared and cubed units do not convert the same as linear units. Through this project, I was able to assess students' demonstrations of curricular competencies, and the students were excited to be doing "real math". Even when students made errors in their calculations, they could still achieve proficient because their numerate thinking was really high level.